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Byzantine Music Intervals: An Experimental Signal Processing Approach

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BYZANTINE MUSIC INTERVALS:
AN EXPERIMENTAL SIGNAL PROCESSING APPROACH

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ABSTRACT

A Byzantine Music piece performed by a well recognized chanter is used in order to derive experimentally the mean frequencies of the first five tones (D – A) of the diatonic scale of Byzantine Music. Then the experimentally derived frequencies are compared with frequencies proposed by two theoretical scales, both representative of traditional Byzantine Music chanting. We found that if a scale is performed by a traditional chanter, it is very close in frequency to the frequencies proposed theoretically, except tone F. An allowed frequency deviation from the mean frequencies for each tone is determined. The concept of allowed deviation is not provided by theory. Comparing our results to the notion of pitch discrimination from psychophysics it is further established that the frequency differences are minute. The Attraction Effect is tested for a secondary tone (E) and the effect is quantified for the first time. The concept of the Attraction Effect has not been explained in theory in terms of frequencies of tones.

CHAPTER 1

INTRODUCTION

Byzantine Ecclesiastical Music (or simply Byzantine Music) refers to the contemporary music used mainly in the Greek Orthodox and the Arabic Orthodox Church. Byzantine Music has its roots in the early Christian centuries. Borrowing the *system*, i.e., the sequence of the intervals in a scale, directly from work done by earlier scholars such as Pythagoras and others, Byzantine Music has evolved throughout the centuries to the present day mainly by means of tradition.

Since the early days of Byzantine Music different musicians and poem prayer writers have attempted to give musical notations that would guide the performers through their task of singing these poems in religious gatherings. Historically, the early Christians would gather together in secret places like catacombs and would chant all together some basic tunes. Most of these tunes would have no melody, let alone harmony, and be sung monophonically with some distinct rhythm, driven mostly by the prosodic intonation of the text. This form of monotonic diction would be a simplified version of what Homer, the ancient epic poet, would use in delivering his poems in public.[*]

As time progressed, various poem writers would write poems that obey a melody based on specific modes used earlier by the ancients, thus standardizing some melodies and poems that would be uttered by all Christians together in the early gatherings. Most of these poems and

[*] Plato often advised “the lyrics should regulate (guide) the music, and not the music the lyrics”.

melodies have survived to this day and are performed primarily in the Greek Orthodox Church in the original ancient language.

Early Christian poem writers aimed to write their poems and texts in general in a clear and common language (Alexandrian common or koine) that everyone would understand. The basic melodies used were meant mainly to aid the uneducated to remember the text rather than to please him acoustically. Through the years, however, these simple, basic melodies evolved into an art form; an art that mostly evolved and progressed during the years of the Byzantine Empire, hence the term Byzantine Music. The word “ecclesiastical” means pertaining to or related to church (<GR ecclesia=church, from ek (from) + kalo (to call)). It is used to distinguish from the music of the Byzantines outside the church, usually referred to as Byzantine Cosmic Music, cosmic meaning “for the people”. In this thesis the term Byzantine Music is used interchangeably with Byzantine Ecclesiastical Music.

Byzantine Music as an art form is now performed by chanters, Orthodox Christians educated in music and language who can render the meaning of the text chanted according to the traditions of the Eastern Orthodox Church. Not only has the body of written genres, essays, poems and theological and philosophical work grown tremendously, but also Byzantine Music is now a complicated art having its own notation and musical scales (*system*), *rhythm*, *tempo*, *kinds*, *modes* and *genders*[*] (Panagiotopoulou, 1981).

It is important to understand the context in which Byzantine Music evolved into this contemporary form of art. Byzantine Music serves the purpose of connecting the faithful to his/her creator according to the traditions and standards of the Eastern Orthodox Church *as defined by the Patriarchate of Constantinople*. Within this context there are specific and well

[*] The above are the “ingredients of melody” as listed in most of the standard theories on Byzantine Music. Whatever information is beyond the scope of this thesis will not be discussed here, but will rather be contained in the referenced citations.

defined restrictions applied by the church to assure that no external elements protrude into the structure of Byzantine Music. This is why, for example, Byzantine Music enjoys a very simple form of harmony which basically consists of a continuous monotonic base tone sounded simultaneously with the melody.* This ancient form of harmony will be discussed later; it is a possible explanation of why higher harmonics seem to possess greater amplitudes.

Another consequence of the purpose that Byzantine Music serves is that Byzantine Music is strictly *vocal*. No instruments are allowed into the church to accompany the chanter's voice. A Byzantine Music choir is assembled by men only, no female chanters are allowed unless the choir is serving an all-female monastery. The fact that Byzantine Music is designed for human voice has an inevitable effect on the way its scales are structured. For centuries instruments have been devised to produce music. Engineering techniques are put to the test every time a new instrument is made, say a new piano or a pipe organ. It is up to the musician to say if the instrument is performing the scales adequately or not.

Throughout the history of music we have seen a plethora of scales that musicians and theorists came up with to solve harmony problems. This is not the case in Byzantine Music, however. The ear and brain (the inspection devises) are embodied in the instrument (the chanter), so that every time there is an inconsistency somewhere in the performance of the scale the voice adjusts its pitches in such a way as to satisfy the ear.

There is an extended literature on how the scales came about. This is a matter of interest to the music historian. The key idea in the history of scales is that at different points in time someone revised an existing scale so that new instrument designs would perform different intervals without unpleasant dissonances. Byzantine Music, on the other hand, is totally vocal, and in effect there are no dissonances to be found in it, unless of course the chanter himself is

* This is called "isokratima" (<GR ison (equal) + krato (to hold)) or simply "ison".

tone-deaf and his ear cannot check up on him and tell him that what his chanting is completely unpleasant!

As a result we are today faced with a group of Byzantine Music scales that have been passed down from generation to generation, from teacher to student for more than two millennia. These scales comprise a variety of intervals, not only the Western tones and semitones. And through the years many scholars, mathematicians, musicians, physicists, etc., have attempted to quantify these various intervals in terms of numbers, i.e., to assign a numerical value to each interval. Accounts of this tendency to ascribe numbers to intervals can be found as far back as 2500 ago; one needs only read through the work of Pythagoras and other ancient scholars in order to convince himself that this idea of quantifying the scales had been taken rather seriously for quite some time now. In this Thesis some background information will be given in order to illuminate the scope of the present research, but nothing will be discussed in depth as there are numerous excellent recourses for this sort of investigation (Backus, 1969, Benade, 1990, Chrisanthou, 1832, Jeans, 1968).

For the rest of this research paper I have decided to use a terminology that will make the text easier to read by English speakers, in terms of Byzantine Music terminology. There is a variety of terms translated into English by different authors of Byzantine Music manuals and theories. Some of them I find counter intuitive and hard to remember. I will give the translations of words instead of simply the transliterations. For example I will say “kind” instead of “eidos” and “mode” instead of “echos”. This way the reader familiar with the original terminology will relate immediately and the reader who is less familiar (or not familiar) will have a term, which even though it does not translate the meaning, is remembered more easily. Definitions necessary to understand the context will be given as needed. I will also try to avoid repeating definitions

where an alternative explanation can be given. Furthermore, some of the sentences given are translated directly from the ancient or modern Greek by the author. No individual references are given for definitions that are found in standard theory handbooks of Byzantine Music (Crisanthou, 1832, Panagiotopoulou, 1981).

In the following subsections I briefly describe some of the theoretical themes of Byzantine Music that are necessary for understanding the remainder of this Thesis. Then I go on to describe the methodology and some necessary information to understand the signal processing analysis of the data.

CHAPTER 2

THEMES

Notation

Byzantine Music has its own notation derived from Greek symbols and alphabetical characters. Individual symbols are called characters and are divided in two categories: the characters of *quantity* and the characters of *quality*. The characters of quantity tell the musician, given an initial pitch, how many notes he should ascend or descend. The characters of quality tell the musician how to get to a specific tone and once he is there how to perform it.

There are notation characters as old as the 5th century AD. The more complicated the melodies became, the more sophisticated the notation became. By the beginning of the 19th century Byzantine Music notation was comprehensive and complete. The notation used prior to 1814 is known as the "old notation" as opposed to the new one in use today. The new notation was proposed by its three founders: Chrisanth, Bishop of Dirrachion (1843); Hourmouzios, the archive-keeper (1840); and Gregorios, the Arch-chanter (1822). The need for a new notation rose primarily because of the complexity of the old notation, which took some 15 years to master, perhaps because it was based on remembering long musical lines. The new notation given by the

three fathers of the new system of notation is much more analytical and enables us to write a variety of new melodies impossible to write with the old notation. On the other hand, many argue that the reason the old notation was so cumbersome and largely based on memory is that by using fixed musical lines the ancient melodies are preserved better.

The new notation, the one used today, takes three to five years to learn adequately. It is much more flexible than the old notation and allows musicians to elaborate and even analyze older manuscripts. How much elaboration should be done, is a subject much too sensitive to chanters and better left for another discussion. Due to the ability of the new notation to write new melodies and due to the dispersion of chanters to different geographical locations (thus being exposed to other musical stimuli), different Byzantine Music schools or waves were formed. One of the main points of disagreement between these schools is the musical scale intervals, the subject of the present research. A sample of the manuscript of the music piece used here is in Appendix A.

Scales, Modes and Intervals

There are four *different* main scales used in Byzantine Music. By different I mean different not in terms of what the frequency of the first tone of the scale is, but different in terms of the sequence of the intervals in each scale. For example, the European scale "C Major" is identical with "D Major" in terms of intervals within the octave. They both follow the interval sequence:

T-T-S-T-T-T-S ,

where T denotes a tone and S denotes a semitone. The same sequence is followed by *any* major scale and the only difference is in terms of the frequency of the first note. If we rearrange the T's and S's in the sequence above we will end up with the European minor scale, which has the following sequence:

T-S-T-T-S-T-T .

It is evident from the above drastically simplified discussion that major and minor (and for that matter any other kind of European scale) consists of tones and semitones.

In Byzantine Music, however, there are no instruments used and to talk of a base line frequency is immaterial. The chanter chooses a convenient frequency to start his singing based upon his vocal expansion range. The sequence of the musical intervals in Byzantine Music defines what is known as *scale*. The word scale in Byzantine Music has a different meaning and is closely related to the system of Byzantine Music, i.e., the sequence of the intervals within the given scale and not the frequency associated with the first tone of the scale. In other words, scale used in the Byzantine Music sense, means only Minor or Major, without any note next to it. Mode, on the other hand, is a set of rules that defines a distinct way of performing a piece of music. One of these rules is that *one* of the scales must be used mainly within a given mode. For example, "major" in European music language means – roughly – "plagios of the fourth" mode in Byzantine Music language.

The four scales used in Byzantine Music are the *diatonic*, the two *chromatic*s, and the *enharmonic*. Each follows a distinct sequence of intervals that include more than tones and semitones. It is beyond the scope of this paper to discuss any other scale (or its intervals) but the diatonic. The diatonic is used to perform the music piece from which we collected our data.

The diatonic scale makes use of three kinds of intervals: the *major* interval, the *minor* interval, and the *least* interval[*]. Here I will abbreviate them as “M” for major, “m” for minor and “l” for least. To be consistent with the definition of a scale given above, we put these intervals in a sequence so that we construct the diatonic scale. The diatonic scale is as follows:

m-l-M-M-m-l-M

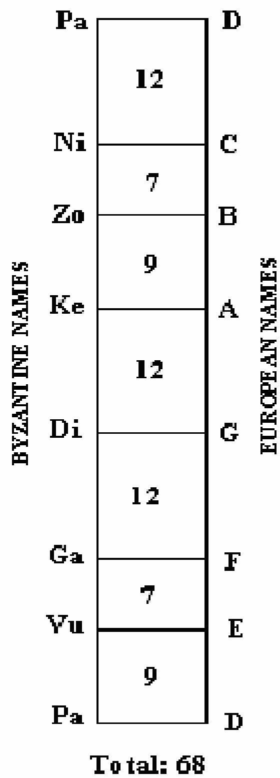
Next we need to define exactly what the intervals are. So far most of the derivations of scale intervals have been based on arithmetical models proposed by musicians and scientists from different disciplines. Personally I strongly believe that assigning numbers to the above intervals is not as necessary as it looks for performing Byzantine Music. In other words, Byzantine Music would survive to this day exactly the way it has even if no one had ascribed numerical values to the intervals. This kind of vocal music is passed from teacher to student verbally for centuries.

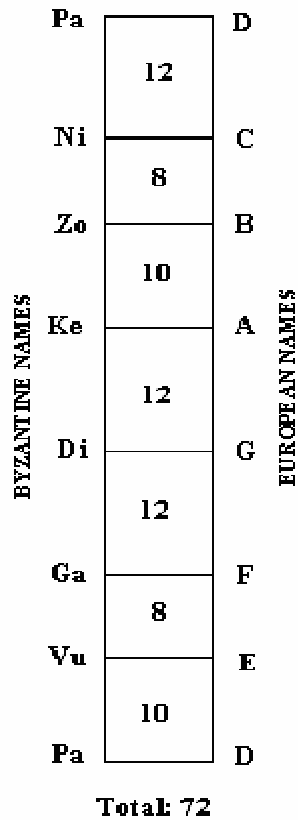
The first theorist to officially assign numerical values to these intervals was Chrisanth, the Bishop of Dirrachion, in 1814.[†] Prior to that publication (Crisanthou, 1832), musicians relied solely on their “good ears” to learn the intervals. After the numerical values were published, virtually nothing changed. Byzantine Music was basically taught the same way after the new notation and after numerical values were assigned to the intervals as it was before. Then why do Byzantine Music theorists consider these numerical values so important? My guess is because of the western influence of the times. Around that time (early 1800’s) many great theorists were dealing with this new hot topic of finding the correct value for tones and semitones, so that European musicians would be able to play more harmonic chords. Even great mathematical minds (Euler) were puzzled by why a particular interval is pleasant or unpleasant to the musical ear. In Byzantine Music we use no chords and no instruments; therefore ascribing this or that

[*] Their transliterations are meizon, elasson, and elahistos, respectively.
[†] There have been some earlier attempts to assign numbers to the musical intervals, but here I will consider only the more formulated derivations suggested during the last two centuries.

value to an interval doesn't really mean much to the young student who learns the scale for the first time. Whatever this young student hears coming out of his teacher's mouth, that is the correct interval between two notes. The number in the book is something highly symbolic and will not change the way chanters perform these scales. This last idea would be of great interest to the psychoacoustician, but is too far from the scope of this paper for discussion here.

Byzantine Music intervals were the center of discussions for a couple of centuries. Many influential theorists gave their suggested intervals and a portion of the Byzantine Music world followed this or that theory. Numerical intervals suggested were based upon a broad range of interpretations. In this paper we will focus our attention on two of the versions of these numerical intervals most accepted today. First we consider an older version proposed by Chrisanth in 1814 and then another proposed by the Patriarchal Committee of 1881. The Patriarchal Committee of 1881 was an official committee appointed by the church to "resolve" any inconsistencies the scales of Chrisanth may have had. It was composed of the most influential musicians of the time and most of them had been trained in European and oriental music, and in Byzantine Music as well. The new scale that was published by the Patriarchal Committee is widely accepted today and is used in many theoretical books as the standard one. These two scales are best understood visually and are drawn to scale in Figure 1 below:





(a) (b)

Figure 1 Two proposed versions for diatonic scale. (a) Chrisanth in 1814 (b) Patriarchal Committee of 1881. Numbers refer to atoms.

From a first glimpse of Figure 1 the reader would notice that (b) is longer than (a). This is a consequence of drawing the scales according to the ratios of each tone to the total length. Furthermore, the scale in (a) is divided in 68 equal parts and (b) is divided into 72. We are going to call these parts *atoms*, because they represent the smallest non-divisible interval. No matter how many atoms from D to D the frequency ratio for one octave is 2:1, i.e., whatever the frequency of D is, the frequency of D an octave above should be double.

A comparison between the scales in figure 1 and the European tone and semitone is in order. Byzantine Music theorists think of the major interval of 12 atoms being the same as the tone of European Music. This was actually officially decided by the Patriarchal Committee of

1881. A semitone would then be 6 atoms. Therefore, the major interval (12) is roughly equivalent to the tone (12) in European Music, the minor is roughly ¾ of a tone and the least is slightly greater than the semitone. Of course the above comparisons are rough estimates, because we need to consider not the numbers themselves alone, but ratios of the numbers to the total number of atoms for both cases.

The minor interval from D to E in scale (a) is ascribed the number 9 and the total length of the scale is 68, therefore for scale (a) a major interval has a ratio of interval to scale of 9/68 or approximately 0.1323. The minor interval from D to E in scale (b) is 10/72 or approximately 0.1388, not very far from 0.1323. Similar ratios can be found for not only the intervals of the scales shown in figure 1, but also for other scales proposed by other Byzantine Music theorists. Values for the Diatonic scale are given in Table 1 for reference.

	Chrisanth 1(a)	**Patriarchal Committee 1(b)**
Major	$\frac{12}{68} \approx 0.1765$	$\frac{12}{72} \approx 0.1666$
Minor	$\frac{9}{68} \approx 0.1323$	$\frac{10}{72} \approx 0.1388$
Least	$\frac{7}{68} \approx 0.1029$	$\frac{8}{72} \approx 0.1111$

Table 1 Interval-to-scale ratio (Atoms).

The scales in figure one are so similar in terms of intervals, that are essentially the same. Why are there so many versions of essentially the same (acoustically) scale then? The above question is better answered with an example that pretty much is representative of most reasons that drove Byzantine Music theorists to revise its scales. Let us consider what led the Patriarchal Committee of 1881 to revise the scale to the one shown in figure 1(b). Remember that at the same time in Europe many physicists and other scientists were trying to solve the problem of dissonance for some intervals (thirds, fourths, fifths), thus coming up with different numerical intervals for the European scale. What became known as the Equal Temperament Scale, which basically consists of 12 semitones of exactly equal frequency ratio, was first calculated by the

French mathematician Mersenne (*Harmonie Universelle*, 1636) and J. S. Bach was the one that standardized it. In a nutshell, if the frequency ratio of a whole octave is universally accepted to be 2:1, then we can construct a scale of 12 semitones and the frequency ratio of a tone to its consecutive would be exactly $\sqrt[12]{2} \approx 1.059463094$. This scale adjustment had a huge effect on the musical world of the eighteenth century as new and easy to play instruments were designed. The Patriarchal Committee of 1881 decided to equate the European tone to what they have known as the major tone that has 12 atoms. Then there are 6 atoms in a semitone and twelve such semitones make up a Byzantine Music scale of 72 atoms. So they decided to add an atom to the minor tone, they made it 10 instead of 9, and another atom to the least tone, which became from a tonal interval of 8 to one of 9 atoms. This way each tetrachord (special kind of a "fourth") was increased by two atoms and with two tetrachords in an octave we have an increase of 4 atoms total.

This scale revision, however, did not contribute much to the way Byzantine Music was taught or performed. The Equal Temperament scale was intended to solve a problem that occurred when music is played with instruments that produce a fixed frequency for a given tone. For the violinist playing without the accompaniment of other instruments or the chanter, who sings without instruments, there is limitation on whether they use the Just scale, the Gipsy scale, the Mean Tone scale, the Equal Temperament scale or any other scale for this matter; what is of essence is for the musical piece to be pleasant to the ear. Of course, the scope of Byzantine Music is not primarily to be pleasant aesthetically, but it is important for any music not to have dissonant intervals within its scales. Here hinges the purpose of this paper: to quantify the scales performed which are pleasant to the ear.

After this brief introduction to the Byzantine Music scale it is meaningful to describe the scope of this paper. Unlike the previous top-down approach, i.e., to find the numerical intervals by arithmetic means, here we attempt to determine the interval another way. Using music pieces performed by a well recognized performer, we try to find the intervals that he uses. This may sound a bit on the simplistic side at a first glance, but recall that the concept of musical acoustics was based primarily upon how pleasant or unpleasant two tones of different frequencies are when played together. In the past people used instruments to accompany their music. Because modern instruments (at least the ones we are accustomed) have limitations, we have often abandoned the aesthetical aspect of a piece of music that is not accompanied by instruments.

From this perspective Byzantine Music is one of the very few forms of unaccompanied music that is still alive and performed today. That is where we are going to find authentic scales performed not according to instrumental restrictions, but according to what we (humans) defined as pleasurable to the ear before we had constrained ourselves in the mold of tones and semitones.

I will conclude this section by quoting a very insightful comment by Sir John James, because I couldn't have summarized it better myself (Science and Music, 1937, p.176). Here the author talks about the Equal Temperament scale, the one upon which European Music is based to this day, and some of its drawbacks: "The pianist and the organist accept this accumulation of lesser evils [that the Equal Temperament scale entails] in order to escape the major evils of badly discordant intervals. But the violinist and the singer are under no such necessity; as each interval comes along, they can make it what they like, and so naturally tend to make it that which gives most pleasure to the ear. Observations shew that the intervals which such performers produce when they are left to themselves differ greatly from those they produce when accompanied by an instrument tuned to equal temperament."

The observations which Sir James Jeans speaks of are not to be found commonly in contemporary literature, but are well-known to musicians. With today's advancements, however, in the field of computing and signal processing, collecting and analyzing such data won't be as cumbersome as it was then. The next two subsections give some information on the Fourier analysis techniques used in this paper and the methodology.

Fourier Analysis Techniques

Acoustical signal processing refers to sound or vibration analysis, which extracts information critical for understanding the physical mechanisms underlying noise and vibration (Malcolm, J., C., 2003). In most cases the physicist is interested in the frequency of a signal, primarily because many sound behaviors such as propagation, emission, diffraction, and transmission are frequency dependent. Not only purely physical aspects of sound have frequency dependence, but also the animal (from insects to humans) sensation of sound is highly depended on frequency. With the increasing importance of media, antennas, underwater acoustics etc. the last few decades, acoustical signal processing has become an important tool for the physicist and the engineer.

In this paper we will make extensive use of the Fourier Transform (FT) and the spectrogram. A *transform* is an equation that, given a signal, can provide a spectrum. An *inverse transform* is any equation that, given the spectrum, can recover back the signal. The two defining equations for a transform and its inverse transform are usually called *transform pair*. In general, there are many ways to define such transforms. Transforms used for theoretical representations and proofs usually require an infinite set of data. For example, Fourier transforms, Laplace

transforms, Fourier series, and z-transforms are widely used in proving theoretical relations in most standard textbooks. For finite data records, however, which are the case in real life applications, only finite transforms can be used. Among them one particular transform is of great importance to the physicist and engineer interested in real representations: the *Discrete Fourier Transform* (DFT). The DFT is based on a z-transform derivation (Crocker J. Malcolm, 2003) and is the most widely used tool for extracting spectra from finite length data. Since it was first introduced (Good, 1951), the DFT gained popularity quickly among the scientific community due to its applicability. Since then a computer algorithm was developed to solve DFT's faster and more efficiently, known as the *Fast Fourier Transforms* (FFT) (…). The computation time for an FFT is substantially shortened especially in applications where long N-point signals need to be processed. In general, computational operations are proportional to $Nlog_2N$ calculations for an FFT as compared to N^2 computations for a DFT.

There are various computer software programs for calculating DFT's. For this paper Matlab® version 5.2.0.3084 with a Signal Processing Toolbox was used. When using software it is important to know the specifications of that software in relation to the application one wishes to perform, in order to avoid unwanted results. Matlab® defines the DFT as follows:

$$X(k) = \sum_{n=1}^{N} x(n) e^{[-i2\pi(k-1)(n-1)/N]} \qquad 1 \le k \le N \qquad (1)$$

$$x(n) = \frac{1}{N} \sum_{k=1}^{N} X(k) e^{[i2\pi(k-1)(n-1)/N]} \qquad 1 \le n \le N \qquad (2)$$

where X(k) is the DFT, calculated with a radix-2 FFT algorithm, and x(n) is the original discrete signal. The above two equations are the transform pair we will use in this paper. Equation (1) is the DFT and equation (2) defines the Inverse Discrete Fourier Transform (IDFT).

The Fourier coefficients a and b associated with the signal x(n):

$$x(n) = a_0 + \sum_{k=1}^{N/2}\left[a(k)\frac{\cos(2\pi k t(n))}{N\Delta t} + b(k)\frac{\sin(2\pi k t(n))}{N\Delta t}\right] \qquad (3)$$

are given by:

$$\begin{aligned} a_0 &= \frac{2X(1)}{N} \\ a(k) &= \frac{2\,\mathrm{Re}\left[X(k+1)\right]}{N} \\ b(k) &= \frac{2\,\mathrm{Im}\left[X(k+1)\right]}{N} \end{aligned} \quad , \qquad (4)$$

where x(n) is the discrete signal sampled with a Δt time spacing. Matlab® uses the DFT in equation (1). A fast radix-2 fast-Fourier transform algorithm is used if the length of X is a power of two. “If the length of X is not a power of two, a slower non-power-of-two algorithm is employed. The above specifications came directly from the software’s help files; we quoted these specifications for reference.

The term 1/N in equation (2) is a normalization factor that in the literature sometimes appears in equation (1) and sometimes in equation (2). The reason is that usually in practical computations of the DFT the signal processor usually needs to multiply his equation by a numerical factor that either adjusts the height of the output graphs, or normalizes to unit area, or normalizes to unity at origin etc. These factors usually are reduced to a single multiplication at the final state (Bracewell, 2000, p. 273).

The equations used by Matlab® are often adapted by electrical engineers and statisticians. The physicist feels more comfortable with another convention that solves the problem of dimensional analysis and numerical outcome all in one. If we use physical units, it can be shown that

$$\frac{1}{N} = \Delta t \cdot \Delta f \qquad , \qquad (5)$$

and the transform pair becomes

$$x(t) = \sum_{k=0}^{N-1} X(f) e^{i2\pi kn/N} \Delta f \qquad 0 \le t \le N\text{-}1 \qquad (6)$$

$$X(f) = \sum_{n=0}^{N-1} x(t) e^{-i2\pi kn/N} \Delta t \qquad 0 \le f \le N\text{-}1 \quad . \quad (7)$$

Now not only the 1/N term is shown implicitly in the equations, but also the units will be consistent. For example, if the original signal had units of volts and Δn had units of time (seconds), then the DFT would have units of volts·sec or volts/Hz. If equation (5) is not incorporated in the transform pair, however, the term 1/N must appear (as in pair (1) – (2)). Here we will accept the analysis done by Matlab®.

Acoustic signals are often classified as either *deterministic* or *random*. A deterministic signal is one that, as the word implies, has some deterministic or stable nature. Signals from periodic processes (rotations of a machine) or transient processes (a loud impact) are usually deterministic. Random signals are those that arise from complex sources of sound and are of no deterministic nature whatsoever (signal from a high speed air flow or a turbulent boundary layer) (Crocker J. Malcolm, 2003). Most real signals, no matter if they originate from an underwater natural source or human voice, contain both deterministic and random components.

Spectrogram is a term used widely to describe a two- dimensional plot of intensity vs. frequency and time. In this paper, since we are interested in frequencies, we will often employ the spectrogram. In most cases we will consider the same spectrogram plotted with different colors to reveal finer details. In other instances we will "zoom in" to take a closer look at the part that interests us more than others.

Since we used Matlab® to compute the spectrogram some clarifications are in order. The Matlab® command for producing a spectrogram is:

[B,F,T] = SPECGRAM(A,NFFT,Fs,WINDOW,NOVERLAP),

where A is the vector to which the algorithm is applied, NFFT is the length of the DFT, WINDOW is any window the experimenter wishes to employ, and NOVERLAP is the percentage of overlap between windows. This function divides a long signal into windows and performs a DFT on each window, storing complex amplitudes in a table in which the columns represent time and the rows represent frequency. We can choose our own axes when it comes to plotting the results according to what kind of information we need to extract from the graph. Just as the DFT is applied to each slice, the window of our choice is applied to each slice. This may lead to potential misinterpretations, though, since windowing has some special effects on the outcome.

Consider for example the rectangle function

$$\mathrm{rect}(n) = \begin{cases} 1 & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

to be the window function (or tapering function) for the infinite sequence x(n) that is the original signal. Then the windowed data discrete function – sequence – [$x_w(n)$] is the multiplication of the signal by the window function

$$x_w(n) = x(n) \cdot \mathrm{rect}(n) \tag{9}$$

After windowing an infinite signal we make the drastic assumption that anything outside the window is zero. The original signal changes from infinite duration to finite (truncation) and it is now modified by the window we chose to apply, in this case the rectangle window. Often the truncation of a signal will yield *Gibbs Oscillations* at the discontinuities of the boundaries or around any rapid change of the transform of the windowed signal. Gibbs oscillations are oscillations (decaying overshoot) about the high and low limits of the truncated function; they amount to 9% of a jump discontinuity [$0.09(f_{(x+)} - f_{(x-)}$], for a continuous-time function. The mathematics of this overshoot is now well understood and was first analyzed by Josiah W. Gibbs (1839-1903). Here we will be concerned about the effect of Gibbs Oscillations.

The convolution theorem states that multiplication in the time domain is equivalent to convolution in the transform domain. Therefore, if we define $X_w(f)$, X(f) and sinc (f) to be the transforms of the discrete functions in equation (9),

$$\begin{aligned} x_w(n) &\xleftrightarrow{\text{DFT}} X_w(f) \\ x(n) &\xleftrightarrow{\text{DFT}} X(f) \\ \mathrm{rect}(n) &\xleftrightarrow{\text{DFT}} \sin c(f) \end{aligned} \quad , \tag{10}$$

where

$$\operatorname{sinc}(f) = \frac{\sin \pi f}{\pi f} \quad , \quad (11)$$

then we can write

$$X_w(f) = X(f) * \operatorname{sinc}(f) \quad , \quad (12)$$

where the symbol [$*$] denotes convolution.

Instead of using the sinc-function in equation (12) we can use what is known as the *digital sinc* or the *Dirichlet kernel* ($D_N(f)$)(Marple,…), which is a scaled DFT for the discrete-time rectangle function:

$$D_N(f) = Te^{-i\pi fT(N-1)} \frac{\sin(\pi fTN)}{\sin(\pi fT)} \quad . \quad (13)$$

This windowed function is a version of the original signal but somewhat distorted. The width of the magnitude of the sharp impulses in the DFT will be broadened by the repeated shape of the window transform. The amplitude of neighboring frequency responses is influenced by the sidelobes around a transform peak (*leakage*). This leakage depends of course on the kind of window one uses, how many windows are used (how many slices the software is going to divide the signal), etc. The number of slices, for example, depends on the window/FFT length and the signals length in the time domain.

One way of determining if the bias factor is negligible or not is the following: Since Matlab® slices the signal and applies the window function and then the DFT on each such slice, we can apply first one window and then another to see the difference between the two. We can

plot the difference as another spectrogram, since we are subtracting the difference of one spectrogram from another. This way we will be able to determine computationally and graphically the Gibbs Oscillation bias factor. If the bias is large we will apply some correction techniques, if it is not we will consider it negligible and no further action will be taken. This will be described below, after we present and explain our spectrograms so that the reader better realizes the outcome. Before we discuss how we interpret these graphs, a brief explanation is in order on how we selected our data.

Scope

As opposed to the earlier attempts of Byzantine Music Theorists (Chrisanth, 1814, 1832, Patriarchal Committee, 1881) to assign numerical values to the intervals of Byzantine Music scales based on mathematical manipulations of the frequency relations, here we attack the subject differently: based on performed pieces we attempt to derive the intervals of the diatonic scale of Byzantine Music. In this paper we will consider the intervals of only one of the scales of Byzantine Music, namely the diatonic. To our knowledge, this is the first attempt to derive frequency intervals from a performed Byzantine Music piece.

Methodology

Particular care was given in selecting the chanter who performed the music piece from which we collected our data. This is important because chanters who learned the music from

teachers outside the Byzantine culture use intervals that are closer to the Equal Tempered scales rather than Byzantine Music scales.

The selected music piece is titled “Kyrie ekekraksa”*, which means “Lord I cried [unto thee]” and was written by a well-known composer, Jacob the Arch-chanter (second half of 18th century). The piece was written in the old notation and then translated to the new notation by the three inventors (see p. 5). The performance took place in Athens in 1975. The supervisor of the recording was Professor of Musicology at the University of Athens, Dr. Grigorios Stathis. The performer is Mr. Thrasivoulos Stanitsas (1910-1987), official Arch-chanter of the Ecumenical Patriarchate of Constantinople for several years. Mr. Stanitsas received his music education within and around Constantinople. He is considered one of the most classic Arch-chanters ever recorded. He is a representative of the old school of chanters (the Constantinople School), the only one that is officially accepted by the Patriarchate. We emphasize this, because although in this paper we do not compare intervals among different schools of Byzantine Music, this would be a significant potential research topic.

We wanted to be able to pinpoint the frequency of tones D to A of the diatonic scale, and also to be able to make some inferences on the frequency deviation from the mean frequency that a chanter is allowed and still be correct. In other words, our samples had to be chosen so that after the appropriate analysis we can determine two factors: mean frequency of each tone, standard deviation from the mean frequency of each tone. This would enable us to define experimentally a frequency with a plus/minus allowed deviation from that mean frequency that a chanter is allowed without escaping from the boundaries of the defined tones.

---

* Transliteration of Greek: “Κύριε ἐκέκραξα”. This and other Byzantine Music pieces may be found in the website of *CYLLOGOS MOUSIKOFILON CON/POLEOS* (Copyright © 2000) at http://www.cmkon.org.

A software was used to literally enable us to cut off parts of music from the music piece. These small snippets are the bulk (steady-state) of a given tone and are usually of small time duration, between 0.1 to 0.3 seconds. For a given tone, say D, we used all these snippets concatenated together to construct a signal which can be manipulated by signal processing means. For all tones analyzed in this paper we used 20 snippets, except for tone E (40 snippets) and tone F (24 snippets).

It is interesting to consider the pitch associated with each tone, i.e., perceived frequency by our ears; in a later section we will refer to some classic psychoacoustic studies. Of course this is something beyond this paper's scope, but it is interesting to see after we determine the frequency intervals, if it is possible for our ears to detect such fine differences and, if it is, is it possible for human voices to perform such slightly different frequency intervals at will.

A performer of a music piece eventually passes through all the notes of a given music scale. Then if we want to see what frequency we should assign to each tone[*] we need to go into the piece and isolate one specific tone at the time, say E, and then concatenate all these E's together, perform an analysis of some sort, and make our inferences from that analysis.[†] These have been our main strategies throughout this paper for every tone of the scale.

The piece was rerecorded to fit the sampling rate needs and other format requirements of our research using a standard PC microphone. Since we are primarily interested in the signal's frequency, filtering was kept to a minimum and no software was used to reduce noise. Care was taken when rerecording the signal to minimize the external noise effects. No further filtering was done to the signal. "Isokratima", i.e., another tone sung by a group of chanters simultaneously with the main melody, was not subtracted from the signal. This should not have an effect on the

[*] The terms "note" and "tone" are used interchangeably in this paper.
[†] Another approach is to analyze each occurrence of a tone separately and then apply statistical analysis to the result.

frequency of the melody (the part that we are interested in) so we left it there. The mean of the data was subtracted to reveal fine broadband details. Generally, even though there are many methods of making the graphs look neater, the data were not processed in any such way.

When a music piece is recorded in a studio, various alterations occur upon digitizing and processing the piece in order to reflect the best audio quality possible. Now is as good a time as any to introduce another idea: pitch is not necessarily the same as frequency. Frequency is the physical term that says how many cycles a sound wave has per second. Pitch on the other hand is the sensation of such a quantitative attribute. It is known from psychoacoustics that pitch is a function of not only frequency, but amplitude and intensity as well (Shower, E., G., and Biddulph, R., 1931). Thus when you manipulate the musical piece to meet some audio standards you alter some physical aspects of the sound. Not only studio quality standards change the signal, but also the format the producer wants to save and use to distribute the musical piece has an effect on the signal. Audio compressions – to make a song that otherwise would take five CD's space to fit in 1/10 of a CD – truncate some of the higher frequencies (> 10,000 Hz), for example.

There is an extensive literature on microphones (Malcom J. Crocker, 1998, PART XVI) and in general computer hardware specifications dealing with analog-to-digital conversion. Usually professional microphones are calibrated from the manufacturer and come with a calibration curve that should be considered when considering the outcome. How sensitive the microphone is, has an effect on mainly the amplitude of the acoustical signal recorded and of course it has some effect on the frequencies of the signal. Since in rerecording the musical piece used for collecting data in this paper I didn't use a professional microphone, we will need to make some calibrations of our own and see how much error we have.

Given better recording equipment and original, not processed data, one would fear less of falling into minor misinterpretations of the outcome. Nevertheless this paper solely looks into relative frequencies within the music piece and not if the performer agrees in frequency with other performers or instruments. Here we attempt to find the ratios of the frequencies themselves that construct the diatonic Byzantine scale. It is like taking a gramophone record that has been recorded for the standard frequency of 78 revolutions/min and playing it at 82.6378 rev/min. The *whole* piece would then sound about a semitone higher (because we multiplied the frequency by 1.05946, the ratio of two adjacent frequencies that differ by a semitone), but it doesn't make any difference to the listener. As long as the *whole* piece is elevated or diminished by a multiplicative factor (not additive) it doesn't make any difference because the ratios among tones stand correct. Acoustically a trained ear may realize that the whole piece is a semitone higher altogether, but this doesn't bother the listener; within that piece all harmonies are respected. This is what we do in this paper, in a way. We don't compare the piece to another recording. We seek the ratios within this recording.

Even though we do not wish to compare this music piece to another, we do need to rely on the graphical outcome in the sense that when the graph says that this tone has a frequency of, say, 440 Hz it really is 440 Hz. Tuning forks can be employed in finding how accurate our recording is. Because the frequency of the tuning fork is known, we can estimate an error of the microphone, hardware and even Matlab® program. Tuning forks are ideal for this kind of experimentation, in that they produce no overtones[*] (given they are not stroked too hard).

Another more straightforward way of estimating error in frequency is to have Matlab® generate a pure tone of some frequency, record it and feed the data in our program to see how the

[*] In this paper we will refer to the normal mode with the lowest eigenfrequency (n=0) as the fundamental mode, i.e., the first harmonic. The second harmonic (n=1) will be referred to as the first overtone, etc. (Introduction to acoustics, …, p.53).

graphs come out. We can run both tuning forks and generated tones and see their differences. One thing to keep in mind is that the pure tone either generated electronically or by a tuning fork, was recorded in the same exact way: Mono recording, 16 bits, and with a sampling frequency of 11025 samples/sec.

The next subsection presents some of these graphs from generated pure tones and tuning forks. These graphs have been used as a rough calibration method of our hardware, software, apparatus, and programs. Because the same kinds of graphs are used to analyze the voice itself, we will spend some time talking about what each graph presents.

Error Measurement

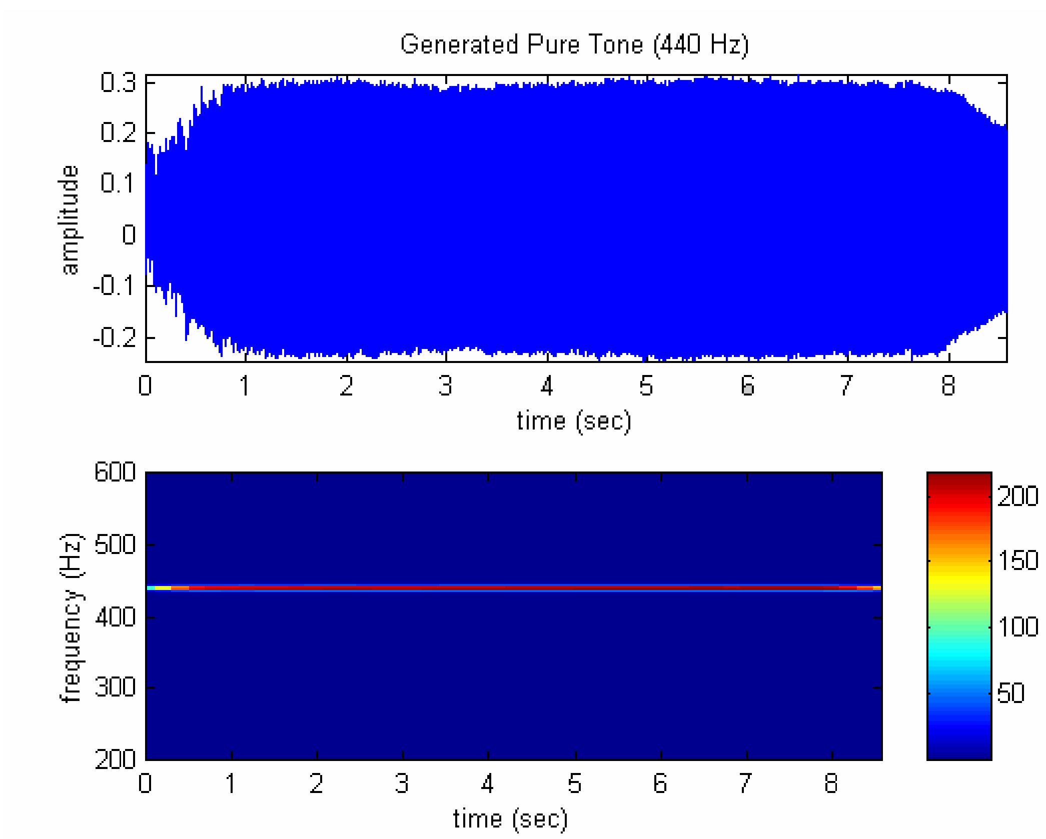


Figure 1-a

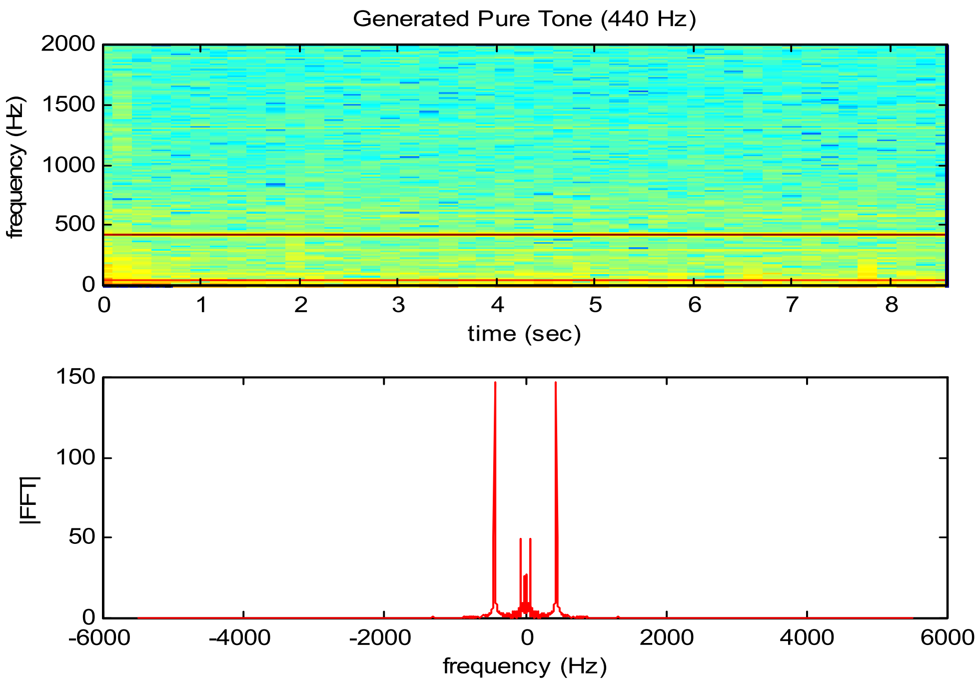


Figure 1-b

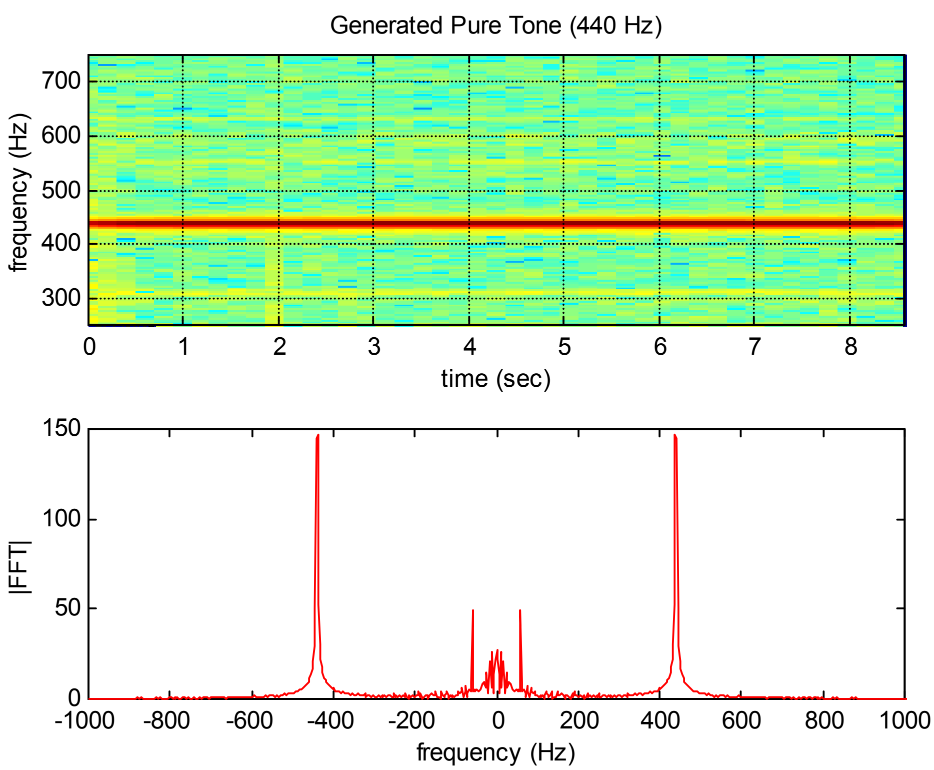


Figure 1-c

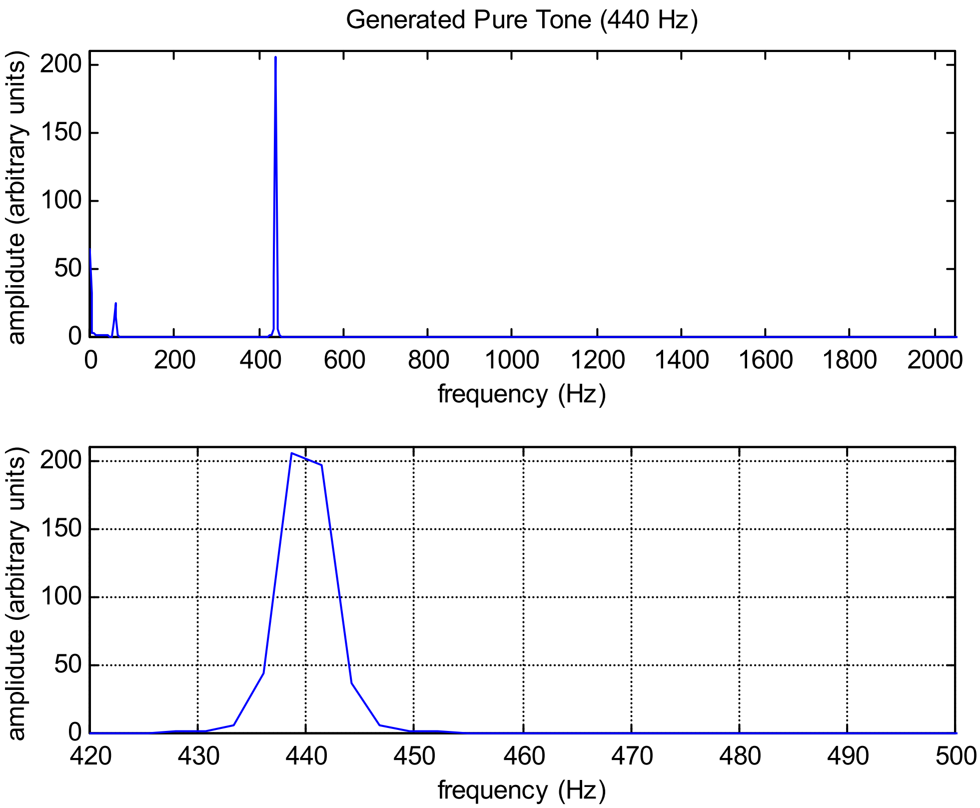


Figure 1-d

This first figure shows the result of a Matlab® generated pure tone of 440 Hz. After the tone was created it was recorded with a microphone. Then we had Matlab® break it down into a data array (94611×1) and we performed the analysis using standard Matlab® commands as well as Signal Processing Toolbox commands.* The sequence of the graphs has been arranged so that some of them are repeated magnified so subtle points can be seen and easily compared with the rest of the graphs of the same group.

Fig. 1-a shows the amplitude vs. time. As we said before, amplitude is of no particular importance when talking about frequencies, but we show it here for reference. The lower panel of fig. 1-a shows the frequency over time, what is sometimes referred to as the spectrogram. This

* The program files (*.m files) used in this paper were mostly prepared and supervised by Dr. Juliette W. Ioup.

plot is of particular importance for us, because it shows how stable a tone is, i.e., how much it fluctuates about the frequency over time. Notice how smooth the pure generated tone is, as expected. The colors indicate amplitude with the "cooler" colors being the lower amplitudes and with the "hotter" colors indicating higher amplitude. On the side there is a scale with an arbitrary units showing the relative high and low amplitudes. We see that, for example, our generated tone did not fluctuate in amplitude, except maybe at the edges, and that was because I intentionally moved the microphone closer and away from the speaker at the beginning and end of the data collection time. As expected amplitude dissipates over distance (inverse square law).

Fig. 1-b shows another spectrogram with different colors. Not only there are no overtones, as expected when dealing with a pure tone, but we can now see some finer details on the spectrogram. We see for example some faint lines close to zero. Again here colors imply amplitudes, so these fine lines below the fundamental are not likely to be hypo-fundamental overtones, but are rather the result of some noise picked up by the microphone of some resonance of the speaker etc.

The lower panel of fig. 1-b shows the FFT (the Fast Discrete Fourier Transform) vs. the frequencies in our signal. Now that we are in the frequency domain we can see where the frequencies happen. We see two abrupt peaks at what appears to be around 440 Hz and some other ones closer to the origin. These smaller peaks closer to the origin are most likely the noise that showed up in the above panel.

Fig. 1-c is the same as fig. 1-b only magnified. Here we zoomed in to see finer details, even though it is not necessary here because our pure tone is so stable; this zoom-in graph will come handy when we need to see other signals that are not so simple in nature. Both graphs now

show the frequency to be at about 440 Hz. It is clearer than the previous graph that considered the whole frequency range.

Fig. 1-d shows amplitude vs. frequency. The upper panel is over the whole length of frequencies and the lower panel is again zoomed in. From this lower panel we can clearly see that the frequency of our tone is indeed 440 Hz. The DC value at the origin (upper panel) is again most likely due to noise, like the same peaks in the previous figures.

The lower panel of fig. 1-d looks as if it doesn't have too many points across its curves; the curve looks kind of rigid. This is a result of windowing length. For example, I chose a window length of 4096 points (N) and a sampling frequency (Fs) of 11025 samples/sec. By equation (5) then $\Delta t \approx 9.0703\times10^{-5}$ seconds and $\Delta f \approx 2.691$ Hz. This means that my resolution (Δf) is the distance on the graph between two adjacent points. The best each point can resolve is Δf. In case I wanted better resolution – more points within the same distance – I should choose a longer length for my window and FFT. Say I choose my N = 32,768 points (a 2^m integer). Then $\Delta f \approx 0.336$ Hz, a much better resolution indeed. There are some drawbacks, however. Not only the time is now longer (2.972 sec as opposed to 0.371 seconds, which is negligible in our case), but certain implications like higher leakage and differences in power may be observed. Not only mathematical differences, but differences in graphs may be seen also. For example, with such a long N, the program is slicing the signal and is applying the window. In other words, each window is so extended in length that is superimposed on the other lines of other windows, and they all look like one straight stable line, which is not the case. It's a leakage-resolution tradeoff that the experimenter has to consider seriously. For this paper we kept the window size as 4096, and this will be the case everywhere, unless otherwise indicated. For comparison the last part of fig. 1-d is repeated with N = 32,768 points this time. Notice that as a consequence the amplitude

has grown with a factor of about ten and the peak is narrower (fig.1-e). Taking more data does give finer resolution of the transform in terms of the width of peaks, whereas padding with zeroes gives finer resolution to the graph of the transform without changing it.

The mainstream method for dealing with undersampling or resolution problems is interpolation, if just increasing the number of points is not desired for some reason. There are numerous software programs that perform midpoint, polynomial interpolation etc. Here we decided that where applicable we will adjust the length appropriately to gain as much resolution possible with sacrificing as little resolution as possible.

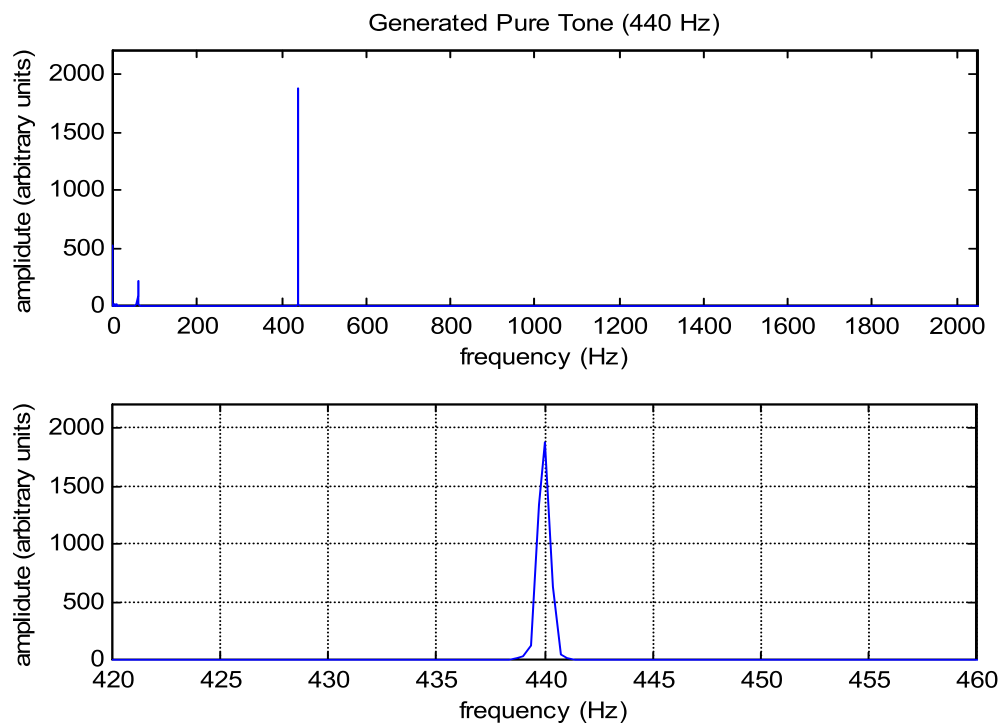


Figure 1-e

Fig. 2 shows the same analysis as was done to a pure tone generated electronically, applied to a tone generated by a tuning fork. Notice the falling amplitude in fig. 2-a. Also notice how the frequency is constant, independently from the amplitude in the subsequent graphs. The last graph shows that the frequency of my tuning fork however is not exactly 440 Hz, but a bit

lower around 437.5 Hz. This is still fine because there are other factors involved in an actual tuning fork experiment, like temperature, tine separation etc. that would affect the outcome. Acoustically a 2 Hz difference doesn't make any difference for the human ear. One can actually hear long low frequency beats when the tuning fork and the speaker produce their 2-Hz-appart tones, indication of some small difference in frequency.

We have also conducted similar simple experiments using other three different tuning forks. Two of them were shown by the graph (a graph similar to 1-e) to be exactly at the correct frequency engraved on the tuning fork, and the third tuning fork analysis showed a frequency of about 1 Hz higher than the frequency indicated by the tuning fork[*].

From a first glance we can see that the problems we may encounter due to equipment are not restrictive for conducting this research. We do have noise and there are ways to eliminate most of these discrepancies, but this is beyond the scope of this paper. For our purposes we are confident that our results, as far as the frequency ratios go, will be trustworthy.

[*] These three tuning forks were provided by the Lab Coordinator N. B. Day.

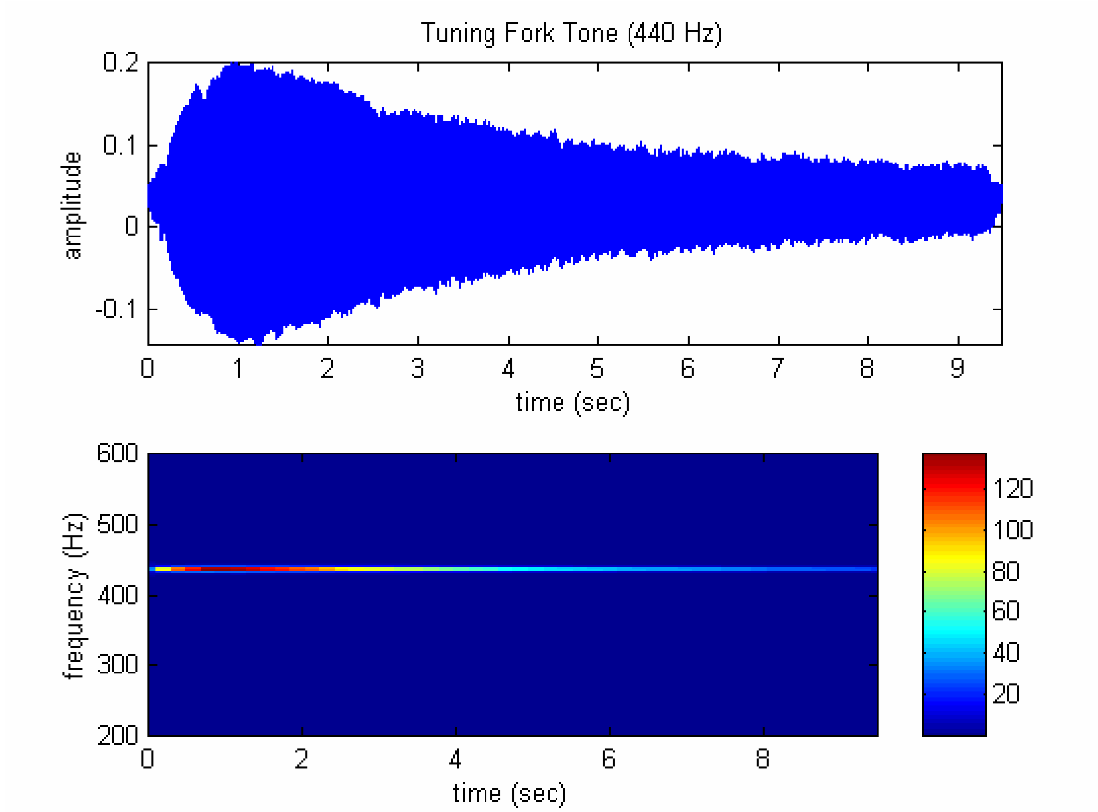


Figure 2-a

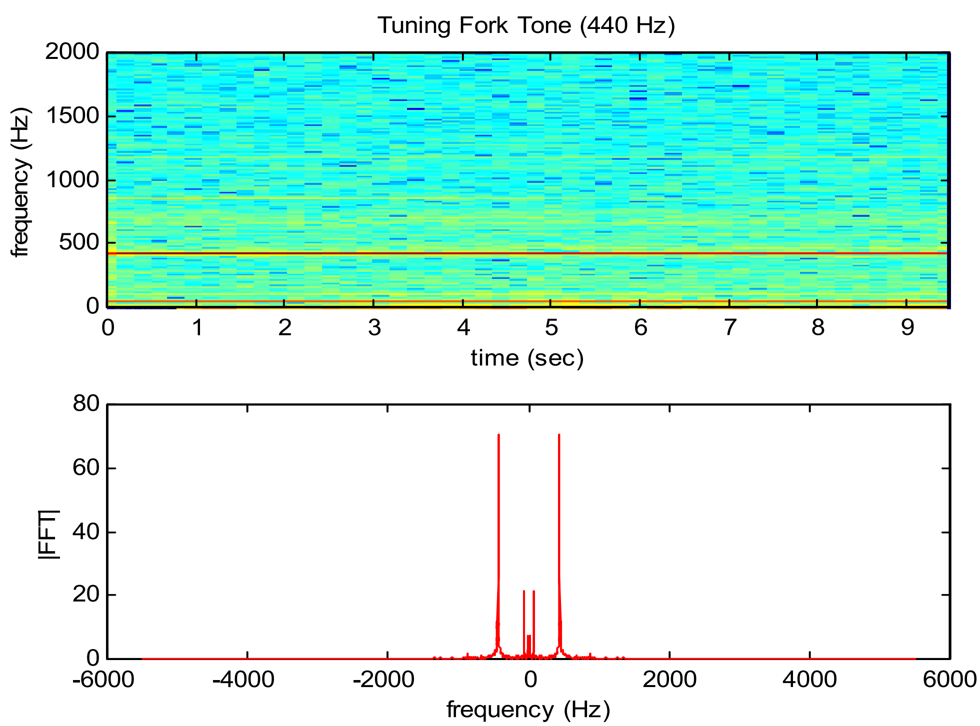


Figure 2-b

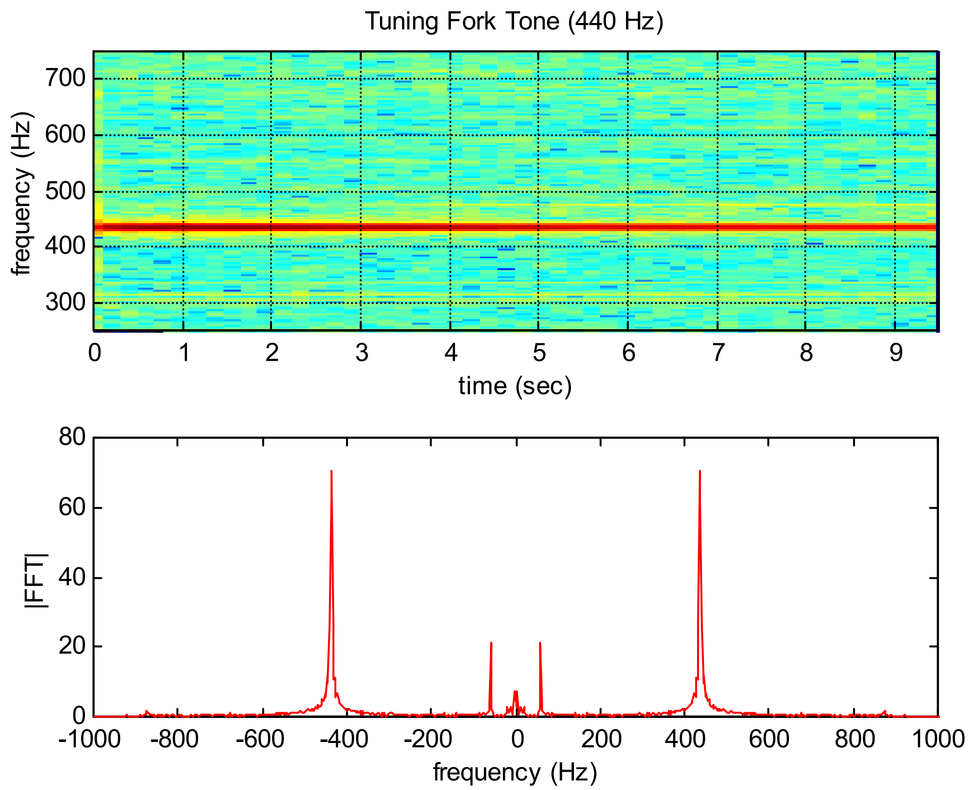


Figure 2-c

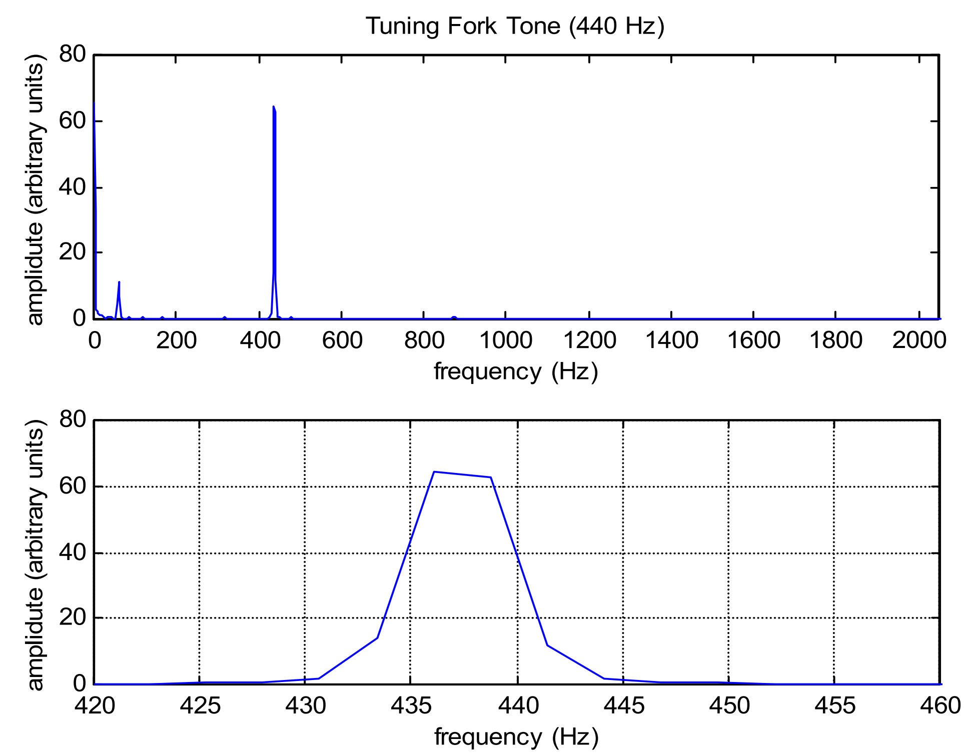


Figure 2-d

CHAPTER 3

RESULTS

In this chapter we present our analyzed data. First we start off with the first tone of the diatonic scale D (see figure 1) and we progressively go up the scale to G, a fourth (tetrachord) above.

As seen in figure 1 above, the first fourth (from D to G) is repeated again from A to the high D. Remember that this is the diatonic scale based on Pythagoras' scale of fifths and fourths. The two fourths are thought to be identical, so if we find the frequency ratios of the low fourth we can generalize this result to the high fourth. Notice the major tonal interval G-A that separates the two fourths; it is called the *disjunctive* tone of the diatonic scale and we will find its ratio later on. Comments on the various graphs are made where necessary for easier interpretation.

The human voice does not behave like a piano string, but more like a violin string that first of all can play more than tones and semitones, and second can produce a voluntary vibrato effect on a tone. Even though the different tones that were extracted from the piece are of short time durations – ranging from 1/10 of a second up to a second – this vibrato effect is still there as expected. We will tolerate an error frequency interval of this vibrato effect.

Another effect when dealing with human voices (especially in performing without instruments) is that tones are not expected to be *exactly* of the same frequency as the chanter

goes up and down the scale. We can call this human error, although I don't like the term personally (even these small variations in frequency have there own musical meaning). I will call it *attraction effect*, i.e., the pull that the main tones of the mode exert on the secondary tones. This is another point this paper will briefly comment on.

In Byzantine Music there are eight modes in total. *Mode* is another word for a set of rules applied to a piece to be written or performed. One of these rules is that a mode must have some *main tones* and some *secondary tones*. The main tones are consonant intervals with respect to the basis tone and the piece gravitates around them constantly. The secondary tones are tones in between main tones and are performed "on your way" to the main tone. So we expect the main tones to be more constant in frequency than the secondary tones. On the other hand, a given secondary tone would be slightly higher if it is performed on the way to a higher tone – if it is between a lower and a higher – and accordingly will be slightly lower in frequency if it is placed in between a higher and a lower tone. For the purpose of this paper D, F, G and A will be considered main tones and E will be the secondary tone.

The higher tetrachord (tones B, C and high D) will not be analyzed for two reasons: first the piece we chose does not contain the high D tone and contains very few B and C tones. We did choose this piece, however, because it contains a plethora of samples for the rest of the tones, especially from tone D to G. It is important to collect all samples from the same piece. Secondly, it is universally accepted that the frequency ratio of an octave is 2:1. It would be nice to check if this is the case, but it is more important to have a sufficient number of sample tones. As we said earlier, the two fourths are thought to be identical, thus we will analyze only the lower tetrachord up to tone G and tone A (disjunctive tone). Then we will assume the higher tetrachord to posses the same frequency ratios for discussion peurposes.

The rest of this chapter will be divided into five sections, each dealing with a tone from D to A. The first section 3-1 discusses tone D.

SECTION 3-1

TONE D

Tone D is the tone where the "isokratima" is held by the group of chanters accompanying the solo chanter. Therefore we expect it to be stable, i.e., without fluctuations about the mean frequency, not only because it is a main tone, but also because of the accompanying chanters.

Tone D is also the basis of the mode that the piece is written in, and therefore the musical piece starts and ends with this tone. For example, we have what is called the *pro-echos*[*], i.e., a constant tone (the basis of the mode D) sung by the chanter at the beginning. This pro-echos has a considerable time length of approximately 3.6 seconds and as a signal it is also continuous and constant in the time domain. At the end of the piece we also have a long tone D known as the *final termination*[†]. The first analysis on tone D will be to compare the first and last D tones (figures 3 and 4, respectively) and see if they agree in frequency. Then we will consider the concatenated D tones (figure 5) taken from the musical piece within pro-echos and final termination.

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[*] Term Translation: Ἀπήχημα ἤ Προήχημα.

[†] Term Translation: Τελικὴ Κατάληξις.

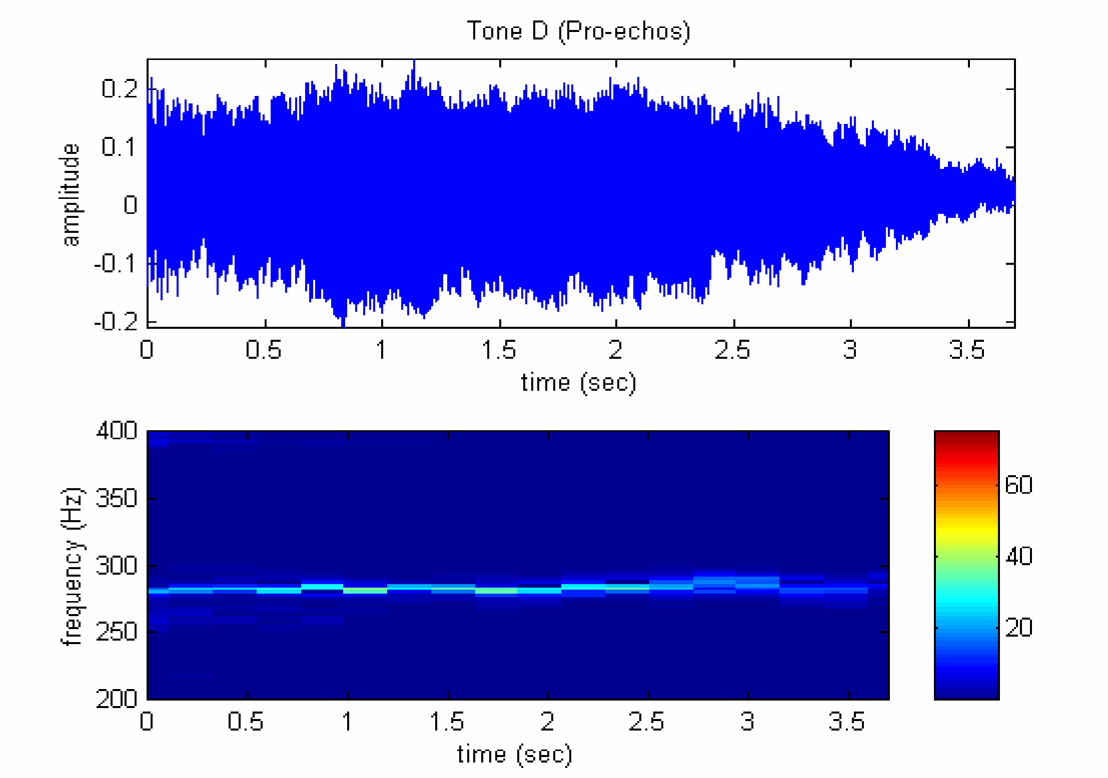

Figure 3-a

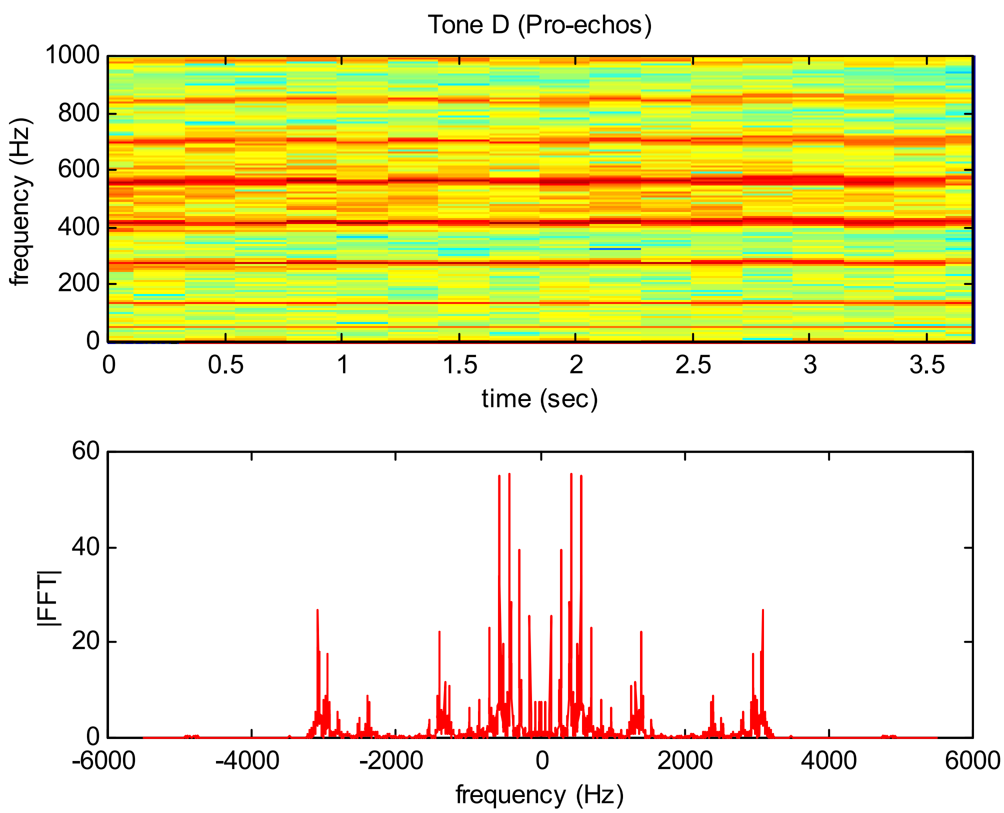

Figure 3-b

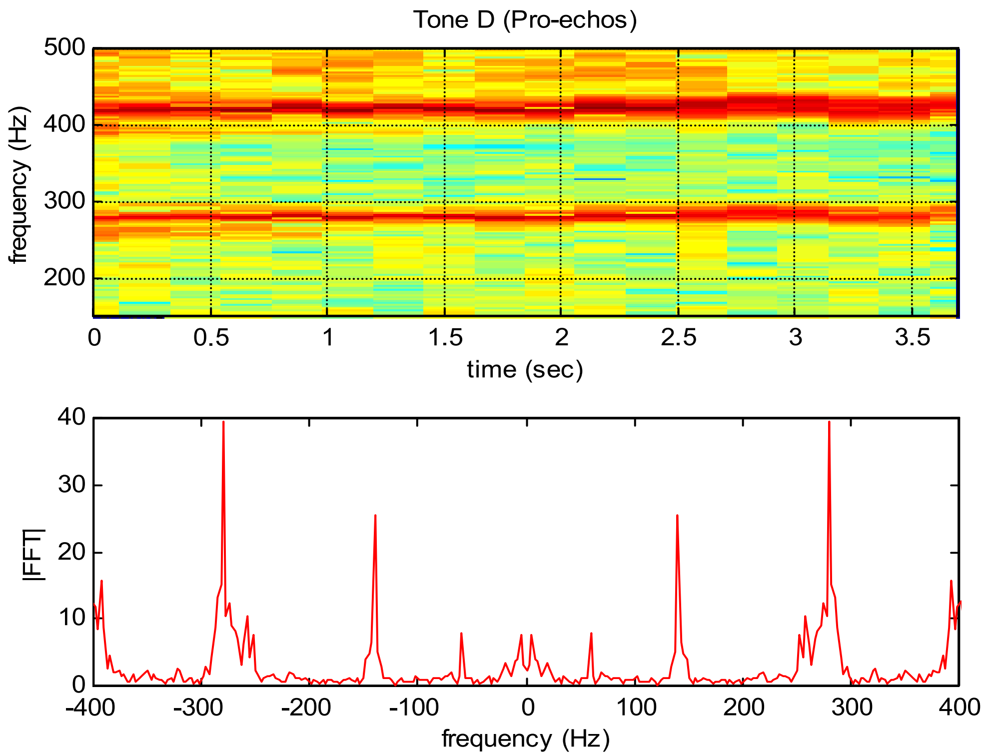

Figure 3-c

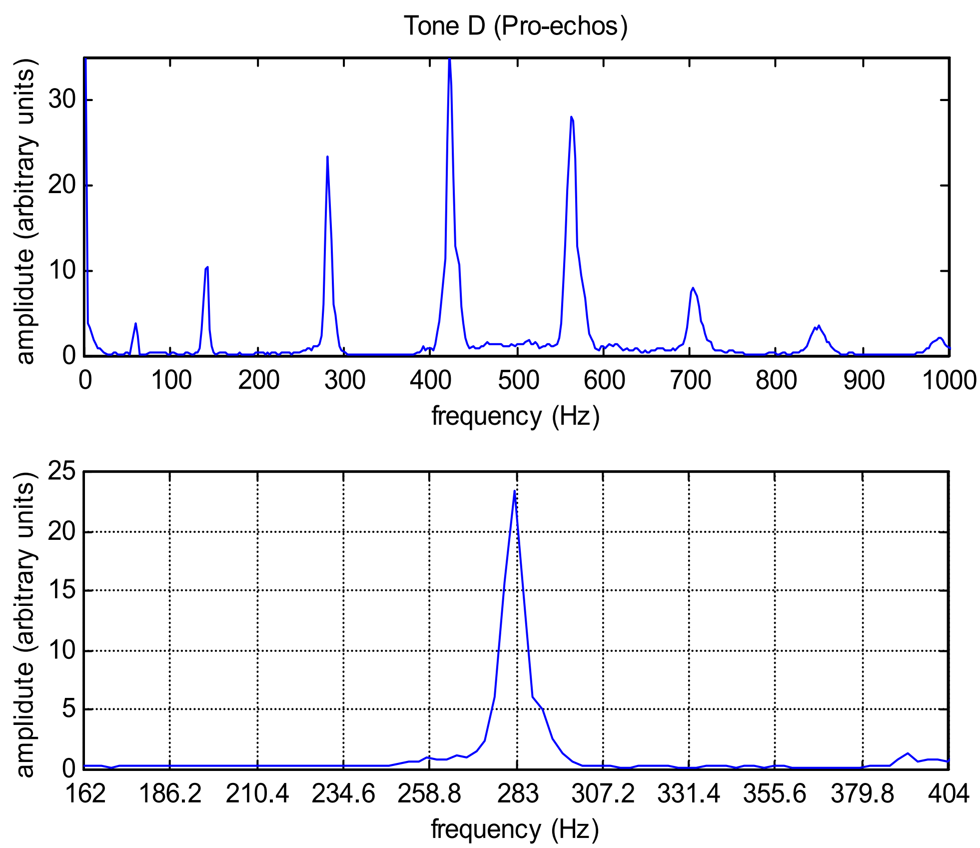

Figure 3-d

Notice the stable frequency in figures 3-a and 3-c, even though the amplitude is growing smaller.* Figure 3-d shows pro-echos to be at 283 Hz. The accepted value for tone D is 293.6 Hz (Jeans, 1968, p.22). Usually chanters, especially at recordings, use a tuning fork that gives the tone A at 440 Hz. Then the tone/basis is found according to the tuning fork, consequently such a frequency difference of the order of 10 Hz is substantial. Let us examine the final termination tone D and see if it agrees with the pro-echos tone D.

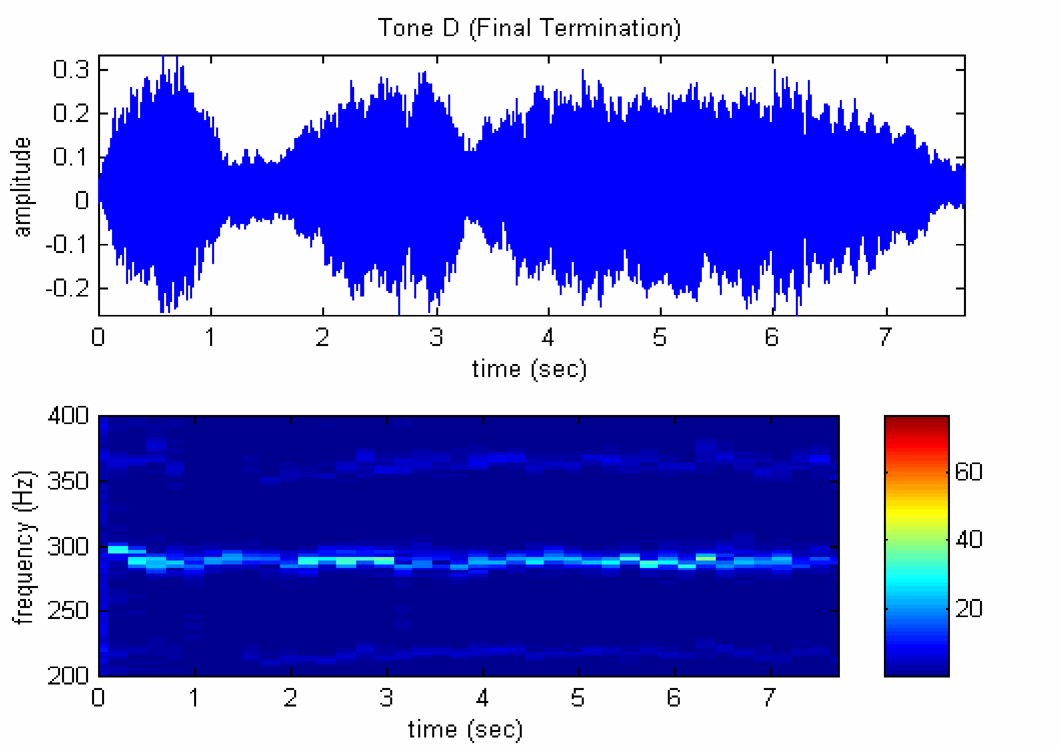

Figure 4-a

* All figures are numbered from a to d and even if we do not comment on some of them, we include them for comparison and reference.

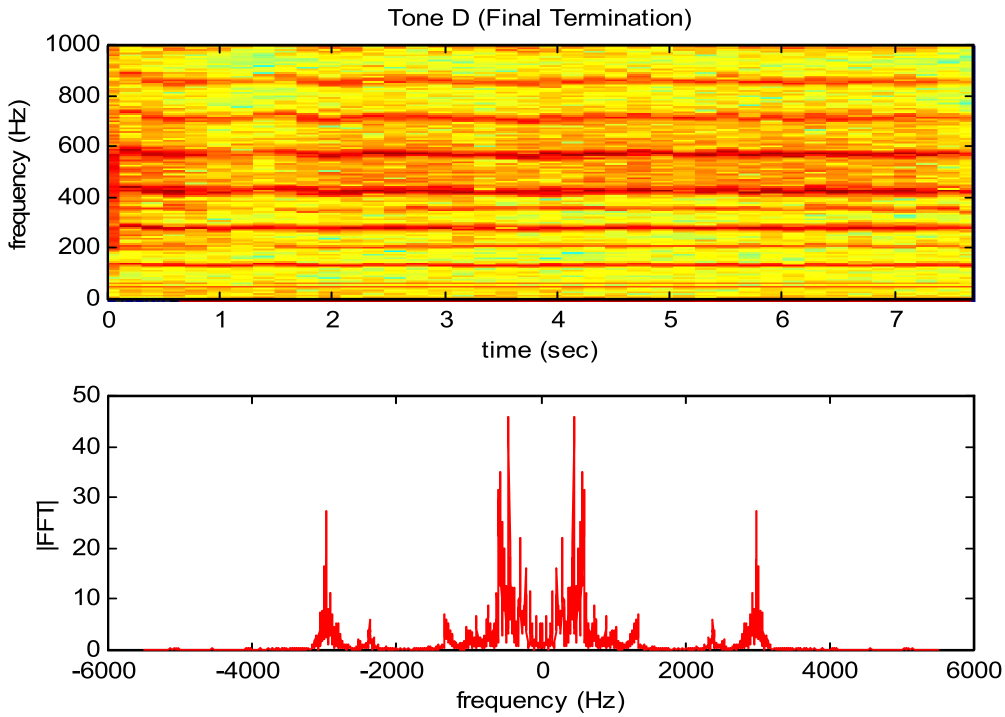

Figure 4-b

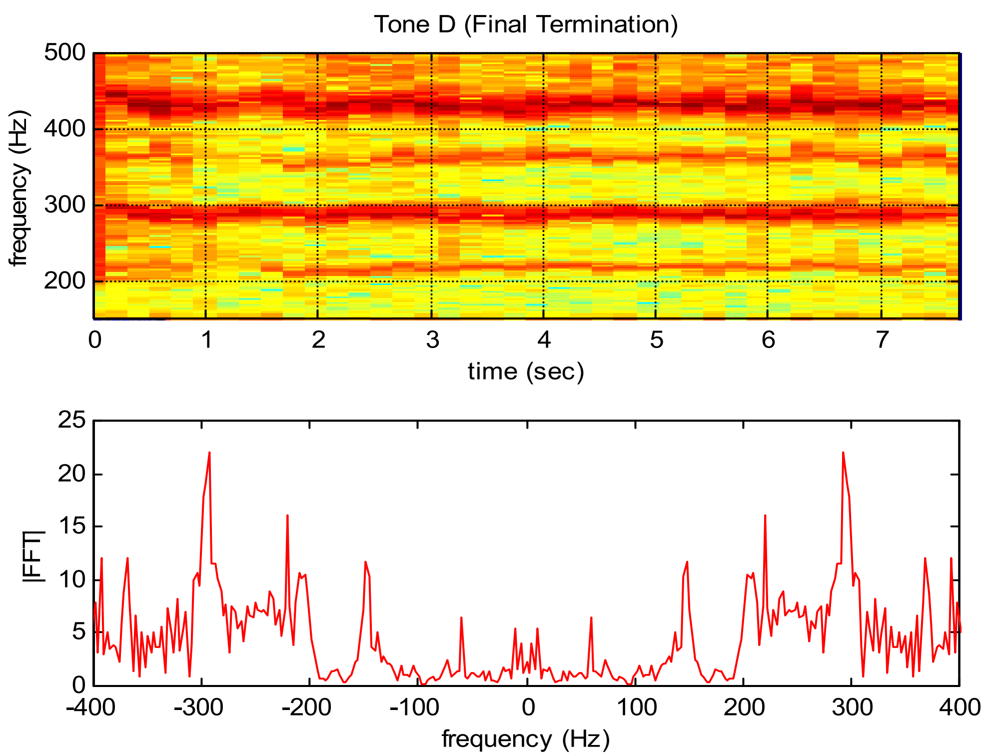

Figure 4-c

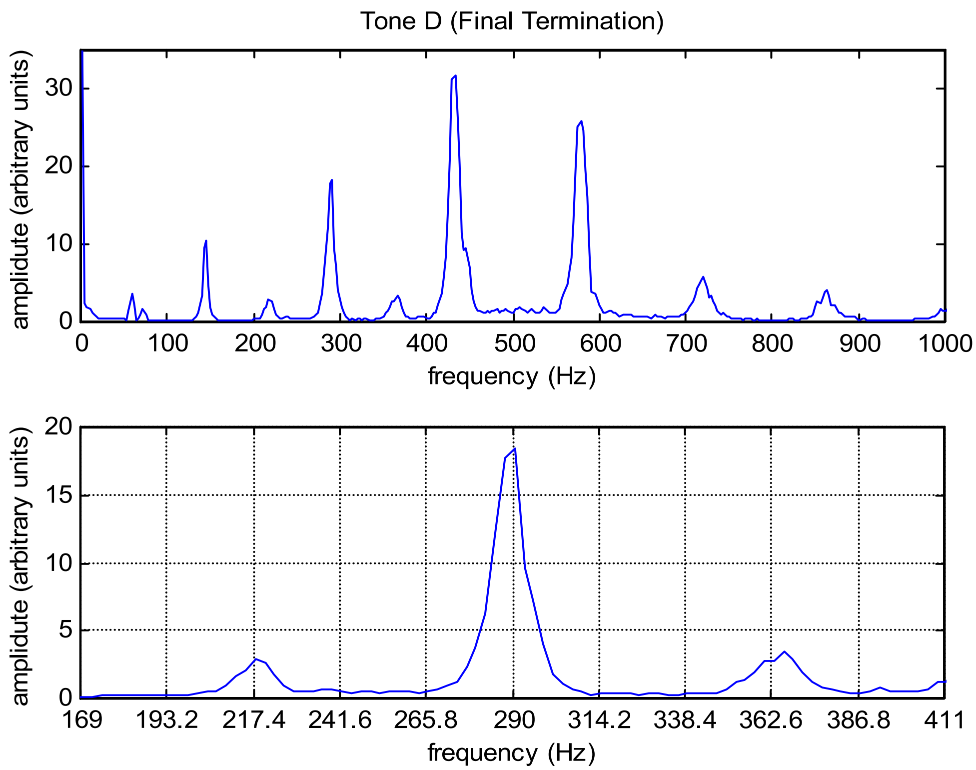

Figure 4-d

The final tone D (figure 4-d) seems to be at 290 Hz, a value much closer to the expected one (292.6 Hz). Now we are faced with the dilemma of which of the two frequencies to accept as the frequency of the tone D. An interesting question, of course, is why the chanter starts with a frequency slightly lower than the one he finishes with. It is beyond the scope of this paper to try to answer questions like that, but an interesting experiment would be to find the correlation of tone memory with the ability to learn music. A possible explanation is that the chanter used his tuning fork to find tone D at around 290 Hz and then because he was left alone for some time to prepare for the recording, he adjusted that frequency to his own vocal needs or maybe he just voluntarily “forgot” the frequency. In our case, however, we do have a last resort, the concatenated D tones; their results are shown in figure 5.

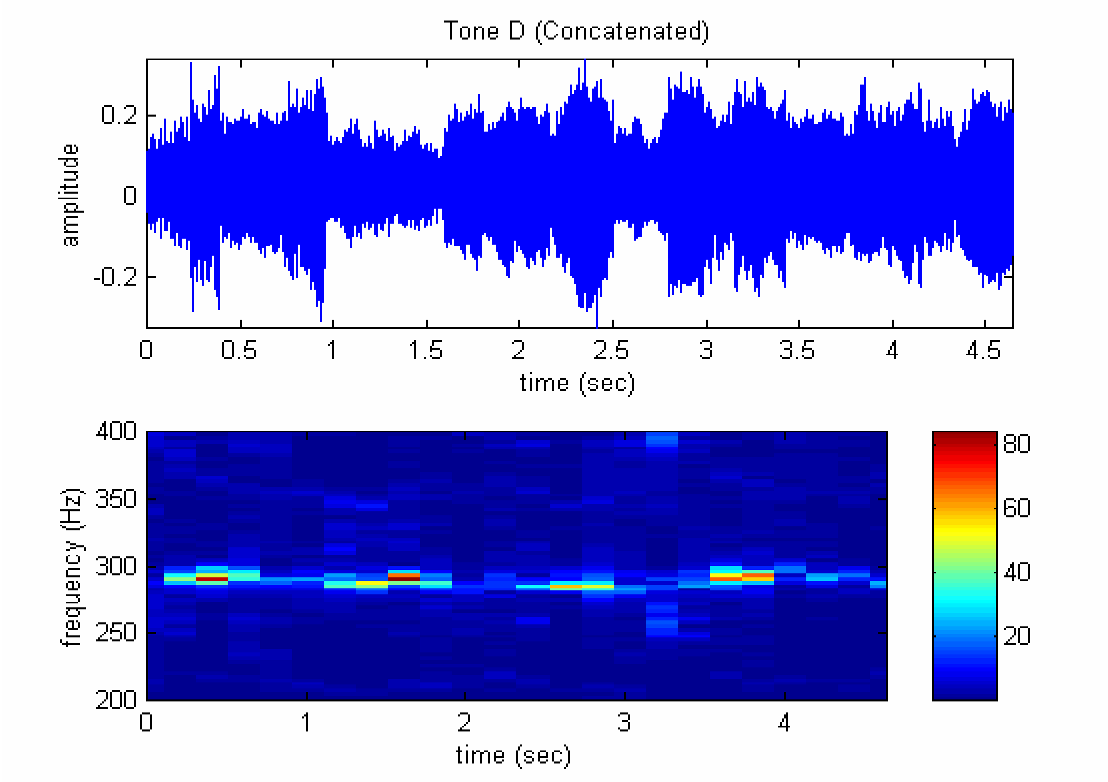

Figure 5-a

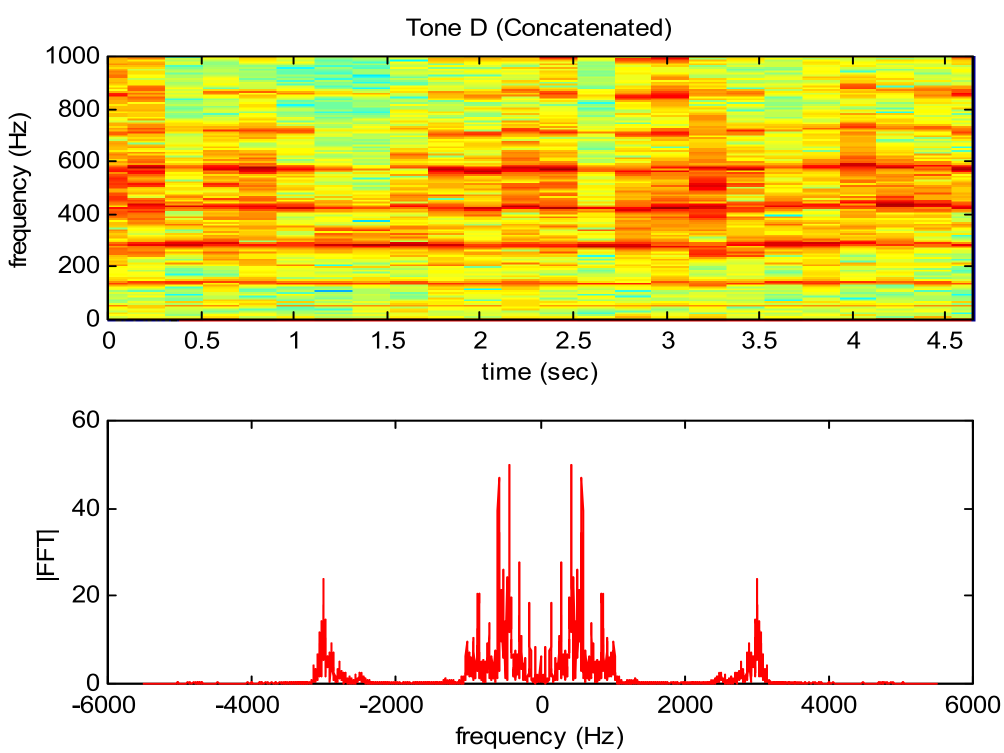

Figure 5-b

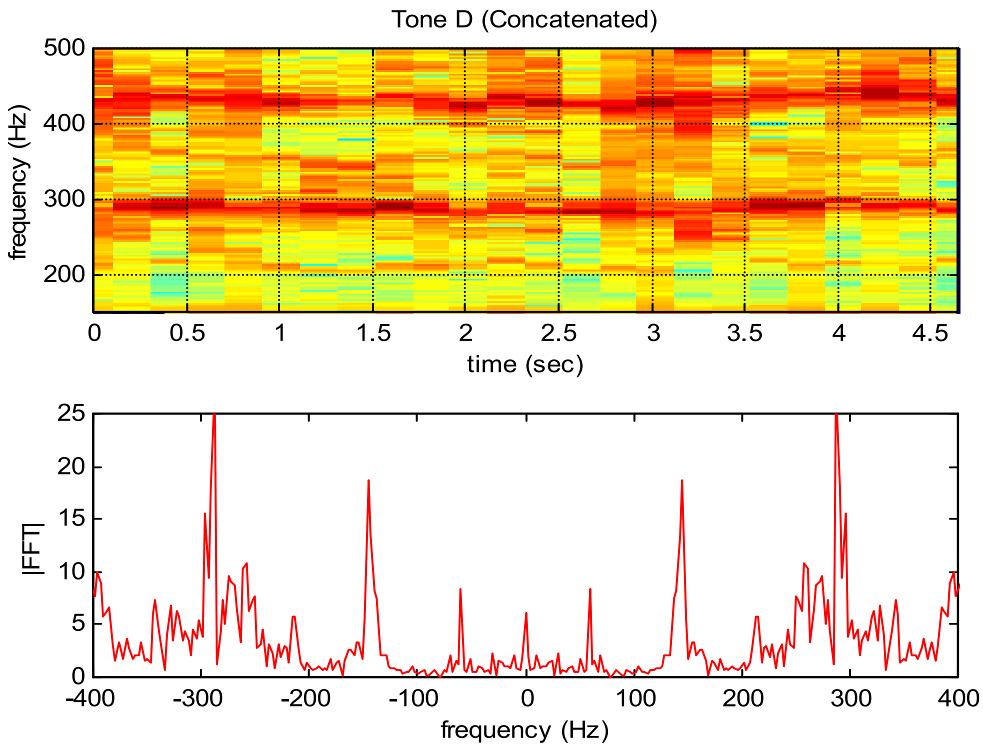

Figure 5-c

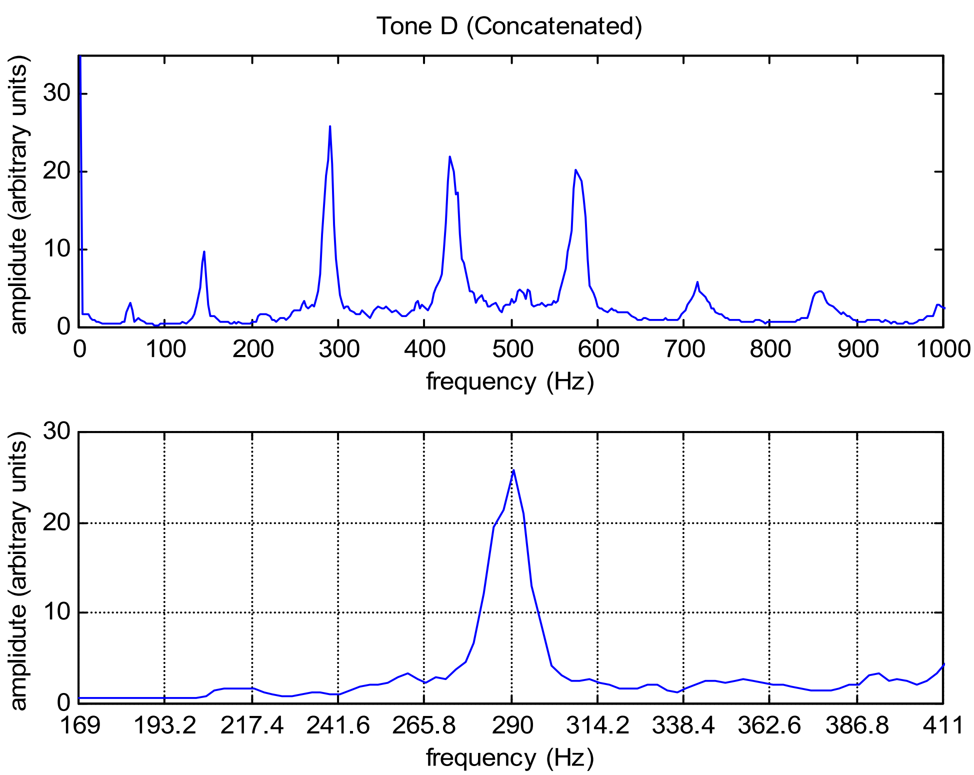

Figure 5-d

Figure 5-a shows the amplitudes of the concatenated notes. Notice that the amplitudes where chosen not to be zero at any time. Also notice the amplitude change, a consequence of a real signal. This is because the singer doesn't always sing with the same amplitude: he sings louder in some instances than others. Figure 5-b shows the spectrogram. In this graph we can see the fundamental (around 290 Hz) and two higher overtones. Surprisingly, some of the overtones seem to have higher amplitude. This is a consequence of what is known as the *formant* of the human voice. The air from the lungs is passing through vocal cords causing them to resonate at some frequency depending mainly on the mass and the tension on the cords. This sound then enters the vocal and nasal cavities before it reaches the mouth (another cavity), which continuously changes shape to produce different vowels and consonants. The final outcome – the voice – is the product of superimposed acoustical waves that resonate in these cavities. As a result, other frequencies are amplified and others are made softer.

Figure 5-d, upper panel, shows what a formant would look like. It is basically the amplitude over frequency plot of a vowel of some frequency (usually a constant frequency). This envelope of amplitudes is what gives the timber of a person's voice. It is the basis of voice recognition software and devices. It is also the reason why we can recognize a familiar voice without seeing the face, e.g., when we talk with somebody on the telephone.

We will often encounter overtones that possess higher amplitudes in this paper. As we said earlier frequency is amplitude independent. On the other hand, amplitude may be higher at some point just because the singer is singing louder at that moment, or because of the formants. We reserve the discussion of pitch and formants for a later section. For the time being we will focus on the frequency of our signal.

From figure 5-b we see that we have overtones as high as about 3000 Hz (lower panel). If there were more overtones in the analogue signal and these were truncated due to studio processing and formatting, we do not know. If there were more overtones and they have been truncated, the richness of the voice is altered. The higher panel shows overtones up to 1000 Hz.

The fact that we deduce the frequency of a tone by looking at the graph instead of using some algorithm to give us a more accurate numerical value may seem unprofessional to some extent. We reserve a more sophisticated method of analysis for future publication. For the time being, we think that the frequency resolution used in this experiment is enough, given the inability of the ear to resolve tones that differ by a small frequency ratio. This subject will be discussed in a later section.

Based on figure 5-d we conclude that the frequency of tone D is 290 Hz. The similarity of the final termination D tone and the 20 concatenated D tones should not be surprising. After all, the frequency difference between pre-echos and the other two tones (~ 7 Hz) is not that big and it can be attributed to other factors. I reserve this topic for future research.

SECTION 3-2

TONE E

Next we proceed with note E of the diatonic scale. Tone E is our only secondary tone, thus we will consider two cases of E: one that the tone E is pulled up and another one that the tone E is pulled down. This attraction effect occurs according to the functional relation between the tones within a mode. Even though we will not discuss the theory of the attraction effect here, we will say that usually an upward pull occurs when a secondary tone is in between a lower and a higher main tone, and a downward pull occurs when a secondary tone is in between a higher and a lower tone. Here all the tone wave files (*.wav) labeled "up" or "dn", for upward or downward pull respectively (see Appendix A), were selected carefully to represent the attraction effect.

Figure 6 shows the graphs of *all* E tones. Figure 7 shows the plots of only these tones from figure 6 that are pulled upward and figure 8 shows the plots of the rest of the tones of figure 5 that are pulled downward.

Figure 6 shows 40 different E tones collected throughout the piece. The first 20 E tones are pulled downwards by note D and the last 20 are pulled upwards by note F. Notice the fluctuation about the frequency in figure 6-a (lower panel). The probable explanation for this fluctuation (not seen in tone D above, or the tuning forks) is that these tones, even though are all E, are performed in slightly different frequencies (attraction effect).

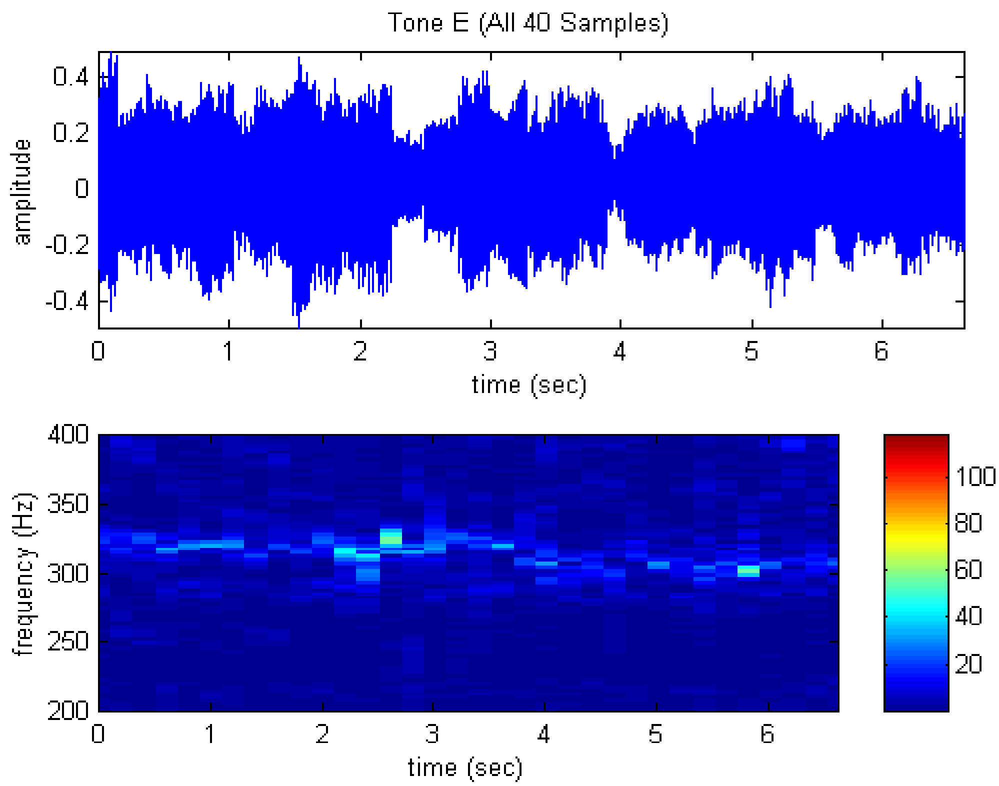

Figure 6-a

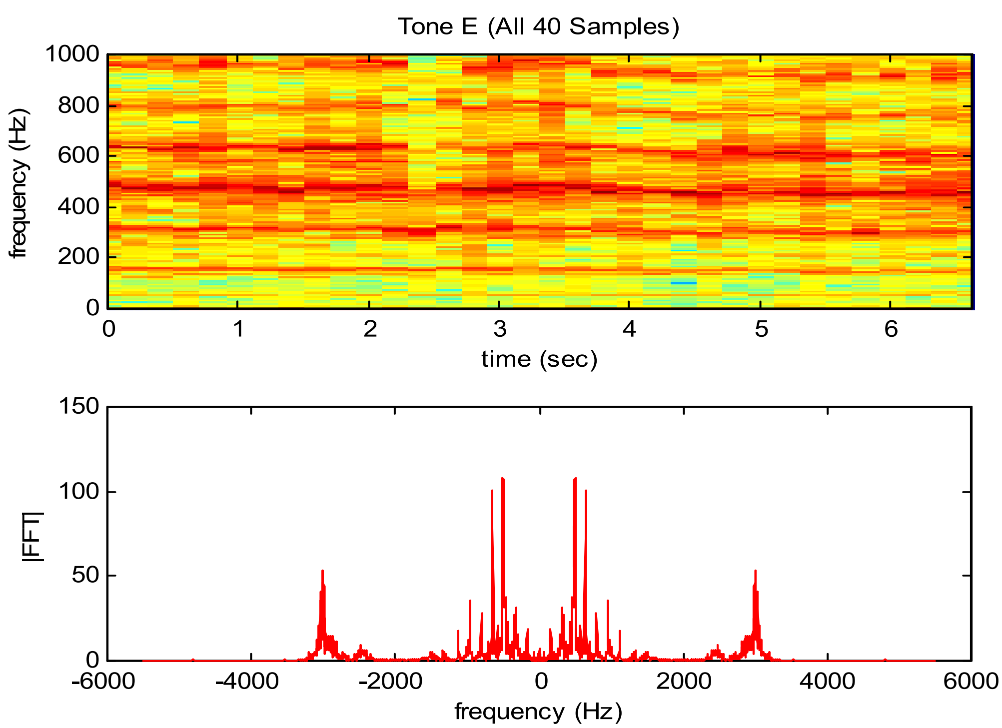

Figure 6-b

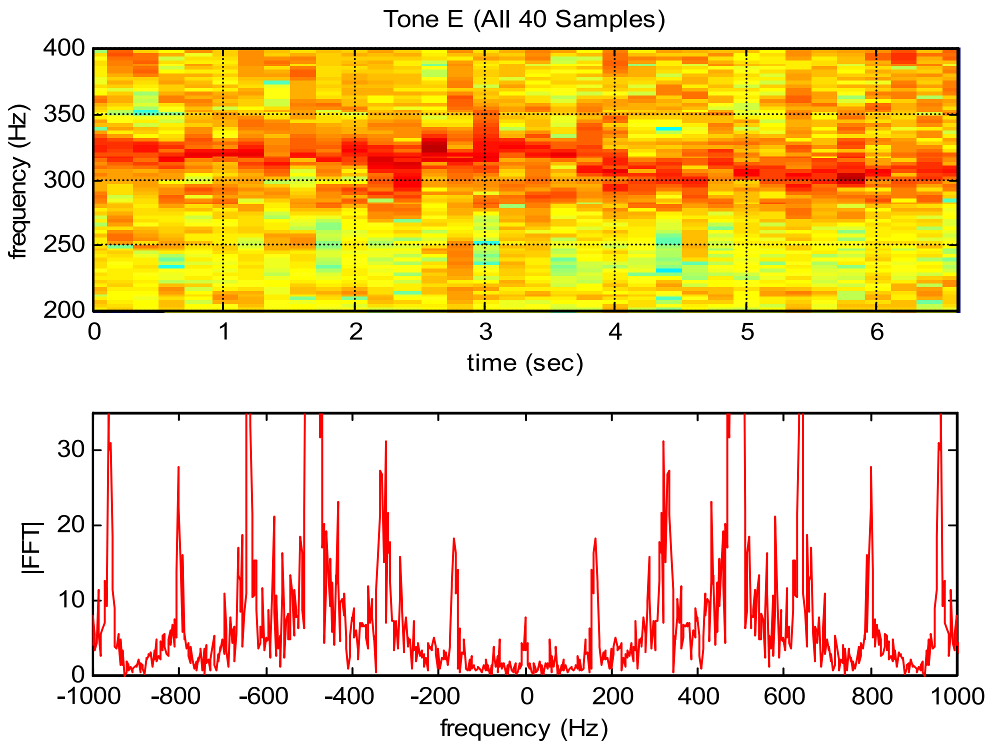

Figure 6-c

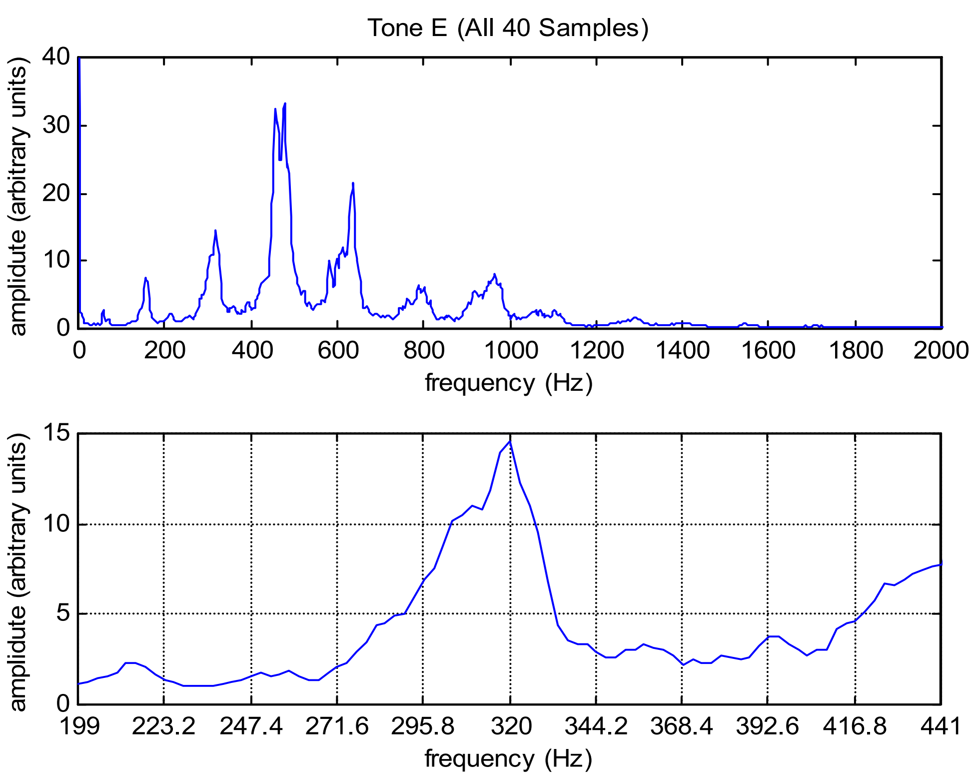

Figure 6-d

The accepted value for E (Jeans, 1968, p.22) is 329.6 Hz. The fundamental must be then the frequency closest to 329.6 Hz, thus we consider the second line in figure 6-b (upper panel) to be the fundamental. A blown-up spectrogram is shown in 6-c. It is easy to see that the first half of the spectrogram is slightly higher than the second one, because the first twenty tones are pulled up in frequency.

Figure 6-d is interesting. It shows the frequencies of the tone as if they were skewed to the right a little. It is not an upright curve like we have seen in D. The peak is on 320 Hz, then we see another local maximum on 310 Hz, and the bulk of the peak falls rather to the lower frequencies. Let us regard the frequency of this tone E as 313 Hz, since the peak is centered on this frequency. Next we will see all the E tones that are pulled up in frequency by F (figure 6).

Figure 7 below shows the E tones of the diatonic scale that are subject to the attraction effect, pulled upwards. Notice how much more stable the spectrogram of the fundamental is in figure 7-c; it almost never touches 300 Hz (horizontal grid line). Also notice how stable all the harmonics are in figure 7-b, compared to 6-b above. The FFT length is still the same, and the axis of the graph are the same also. The stable nature of the frequencies on all the pulled up E tones hinges in the fact that now we are comparing similar tones, as opposed to figure 6 that was considering both kinds of E tones.

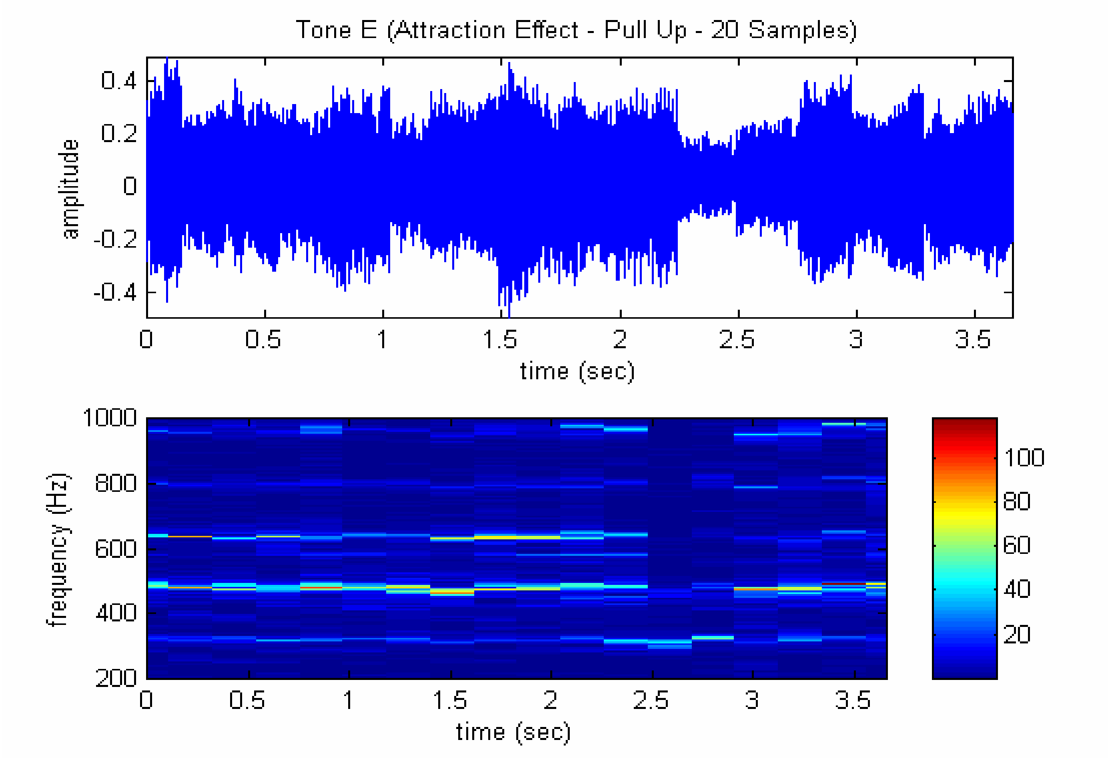

Figure 7-a

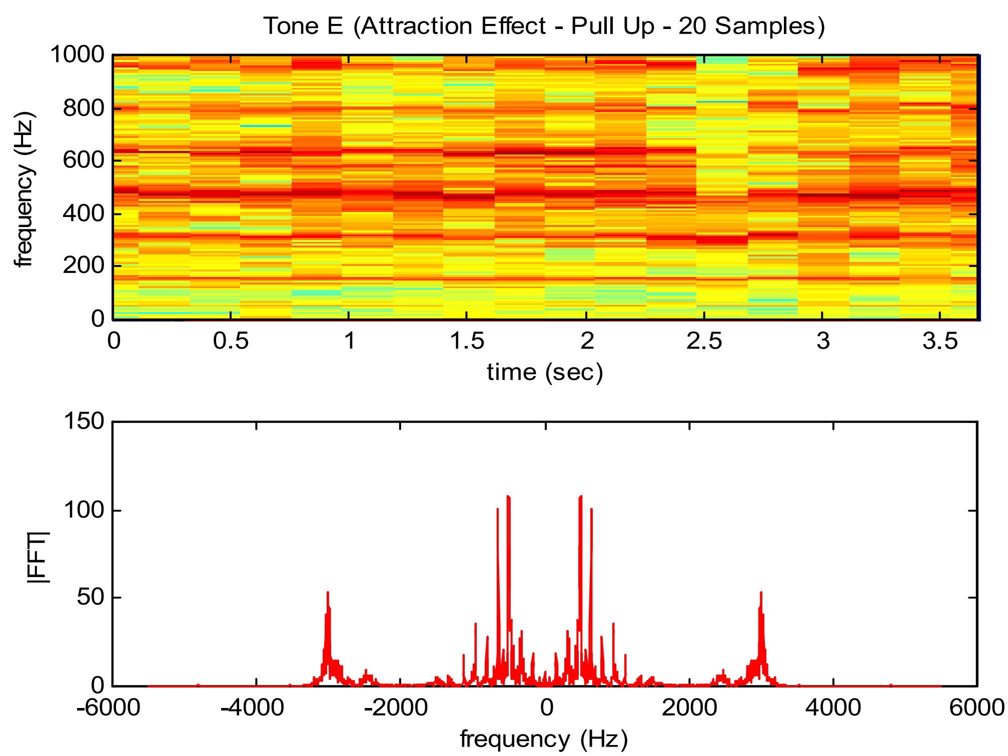

Figure 7-b

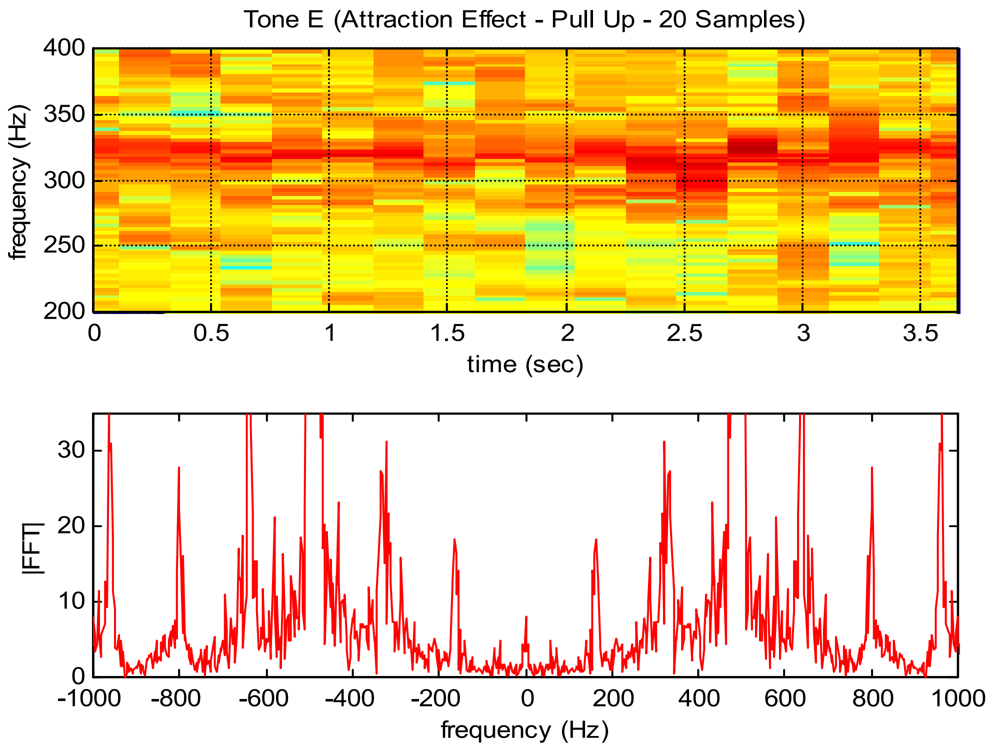

Figure 7-c

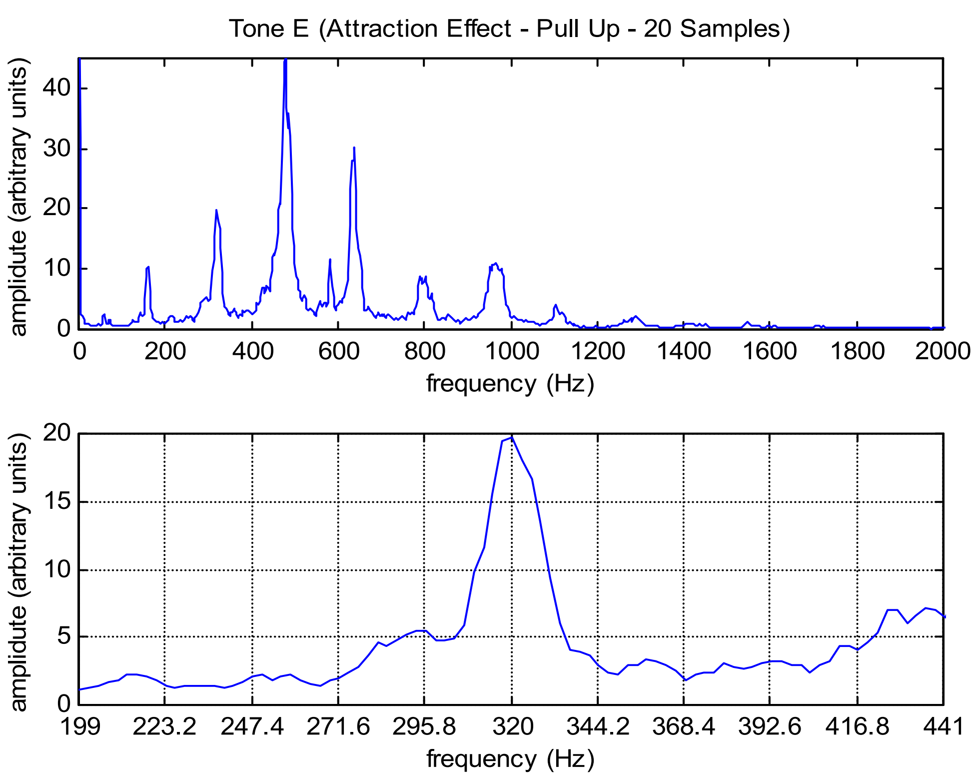

Figure 7-d

In other words, we are faced with two kinds of the same tone E. Some middle ages pianos and organs had additional keys to play these intermediate notes, but they were abandoned due to difficulty in performing such “over-keyed” instruments. This is one of the points this paper wants to make: voice performs without instrumental restrictions, yet no dissonance is present. There can be consonant intervals other than the equal tempered intervals. Byzantine Music takes advantage of its non-instrumental nature and to measure and report these intervals experimentally is important.

Figure 6-d again shows the format (upper panel) of tone E with different vowels, hence the difference in amplitude over the harmonics. The lower panel of figure 6-d shows an upright, clear frequency of about 320 Hz for the fundamental.

Next we consider all the E tones that are pulled down by D (figure 8).

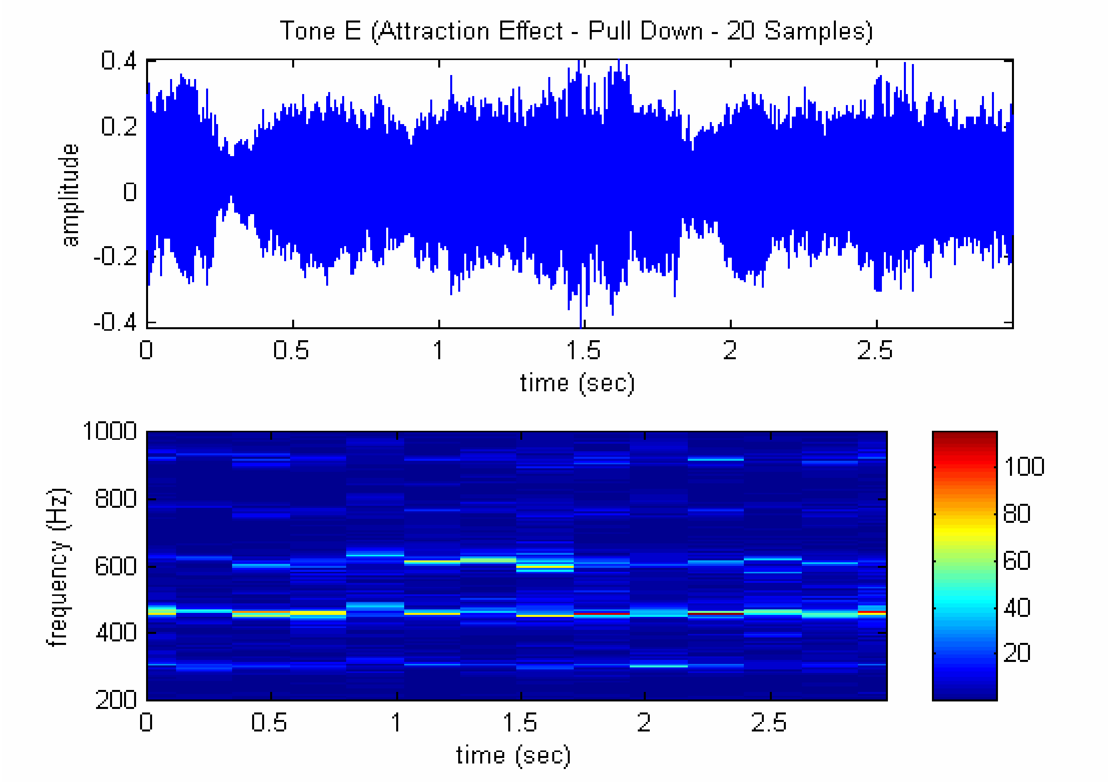

Figure 8-a

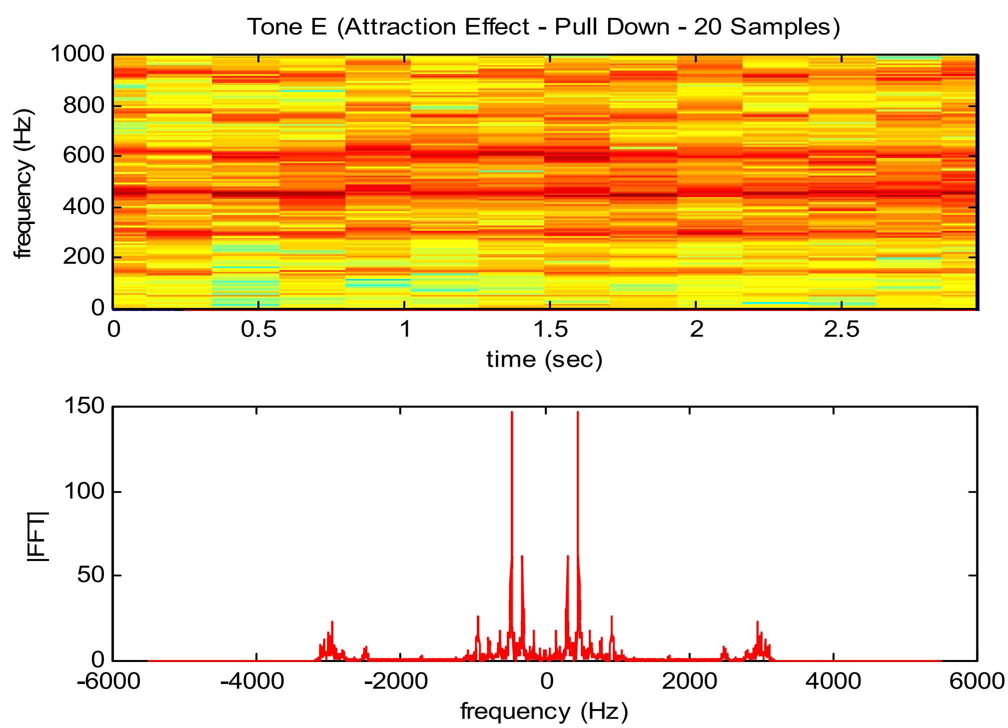

Figure 8-b

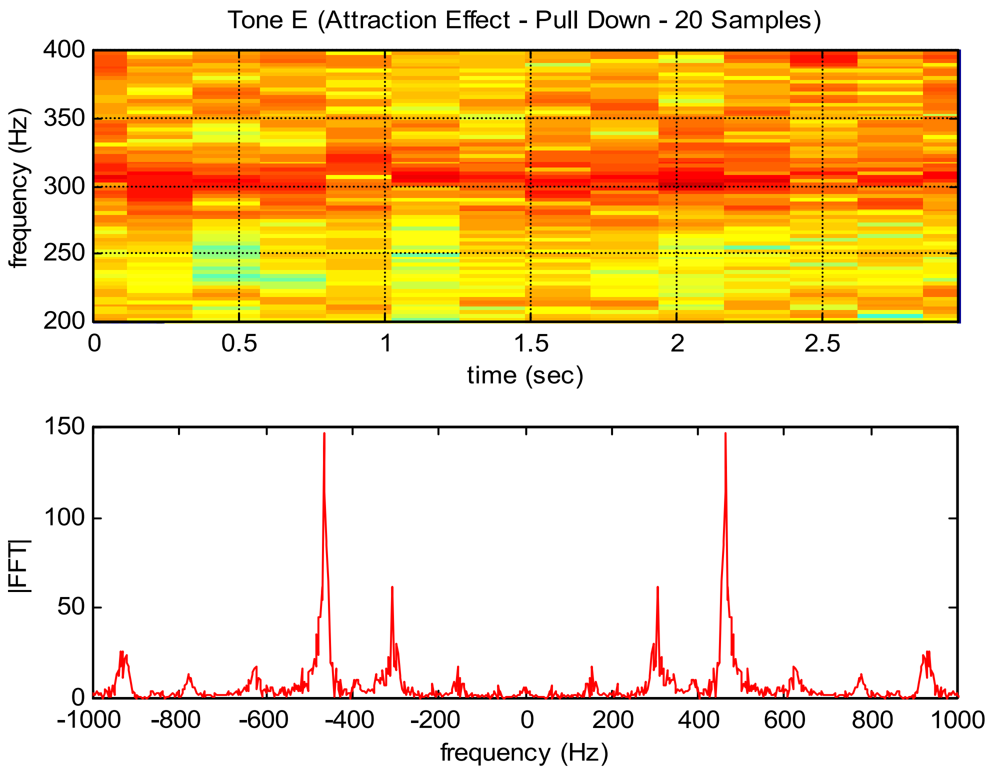

Figure 8-c

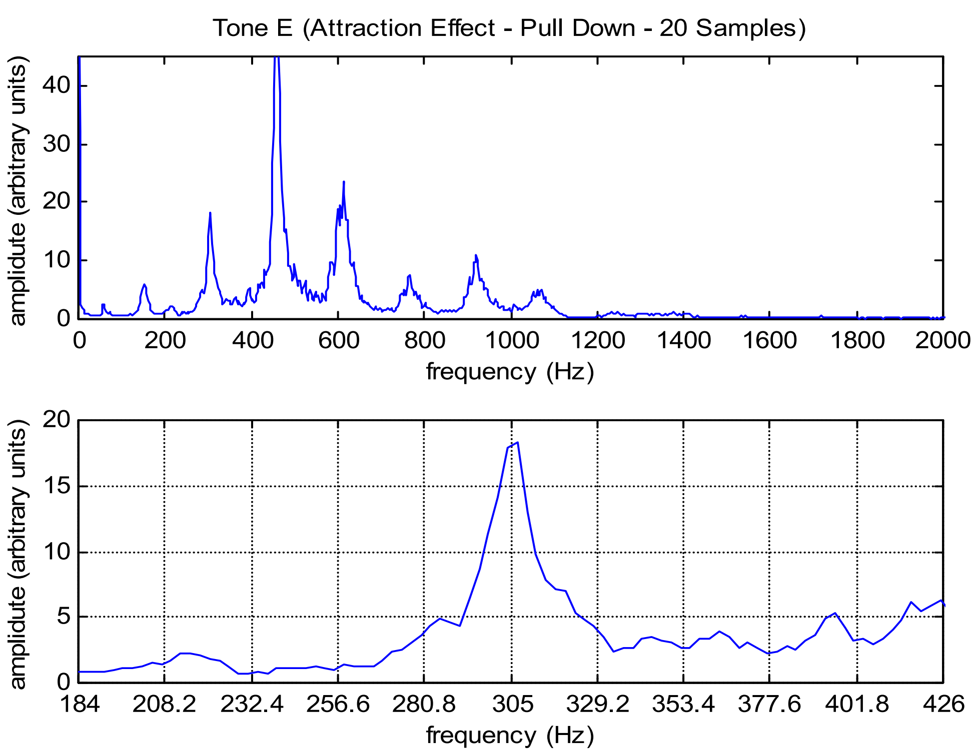

Figure 8-d

Figure 8 is the result of all the E tones that are pulled down by D. It was the most tedious tone to collect, because the singer usually voices this particular tone with a hyphenated leaning towards D. The total time length of this signal is approximately 3 seconds, the shortest of all signals in this paper. The number of tones is 20, the same as in the E tones pulled upward.

In figure 8-a and 8-c we see how stable the signal is about the mean if we compare it to the one in figure 6. Figure 8-c shows the signal slightly lower (~ 300 Hz) than the one in figure 7.

The graph of interest is the one in figure 8-d, where we see again the frequency of the signal (lower panel). It seems not to be far from 305 Hz, some 15 Hz lower to the same E tone when pulled upwards.

Table 2 summarizes section 3-2. It shows the frequencies of tone E pulled up by F or down by D and it also shows the frequency of the tone when both up and down pulled E tones are considered.

	Frequency (Hz)
Pulled Up	320
Pulled down	305
Pulled up and Down	313

Table 2. Tone E

SECTION 3-3

TONE F

Next we supply the data analysis for tone F. Tone F is a main tone, therefore we do not expect it to be subject to the attraction effect. Nevertheless, we collected F tones that were in between E and G and tones that were in between G and E. In other words, we followed the same procedure for collecting the attracted tones for E (the secondary tone) and we subjected them to the same analysis to see if the attraction effect takes place in the case of the F tone.

The results are shown in figures 8, 9 and 10 below. Figure 8 shows all 24 samples containing the possibly pulled up and the possibly pulled down F tones. Figure 9 shows only 12 samples of pulled up F tones and figure 10 shows only 12 samples of possibly pulled down F tones.

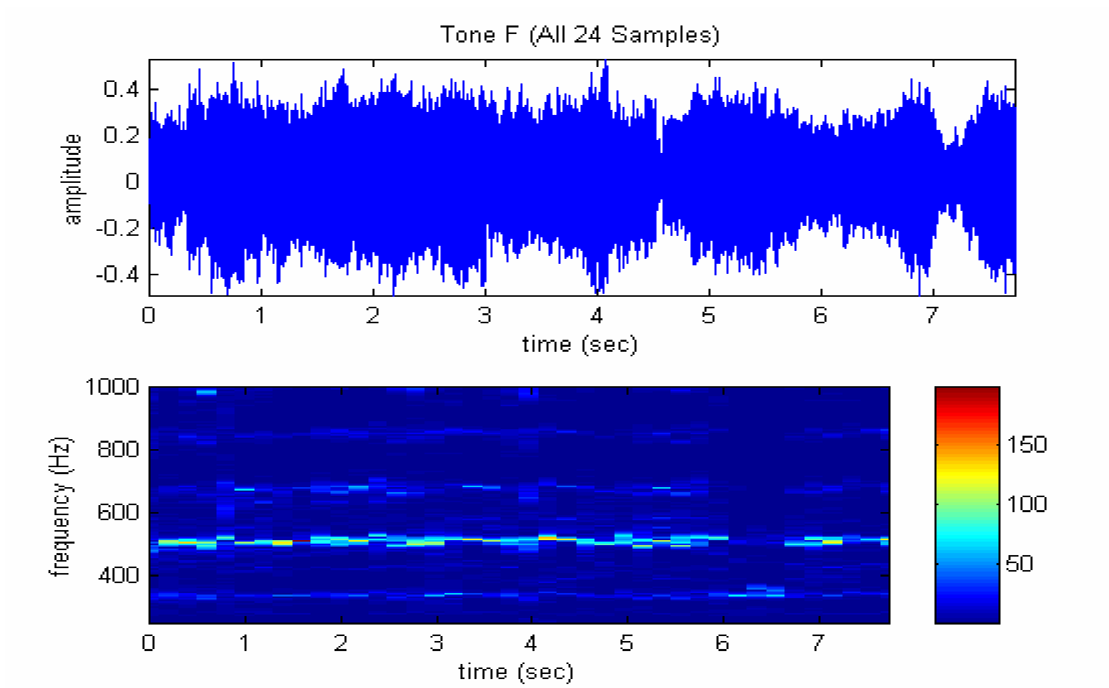

Figure 8-a

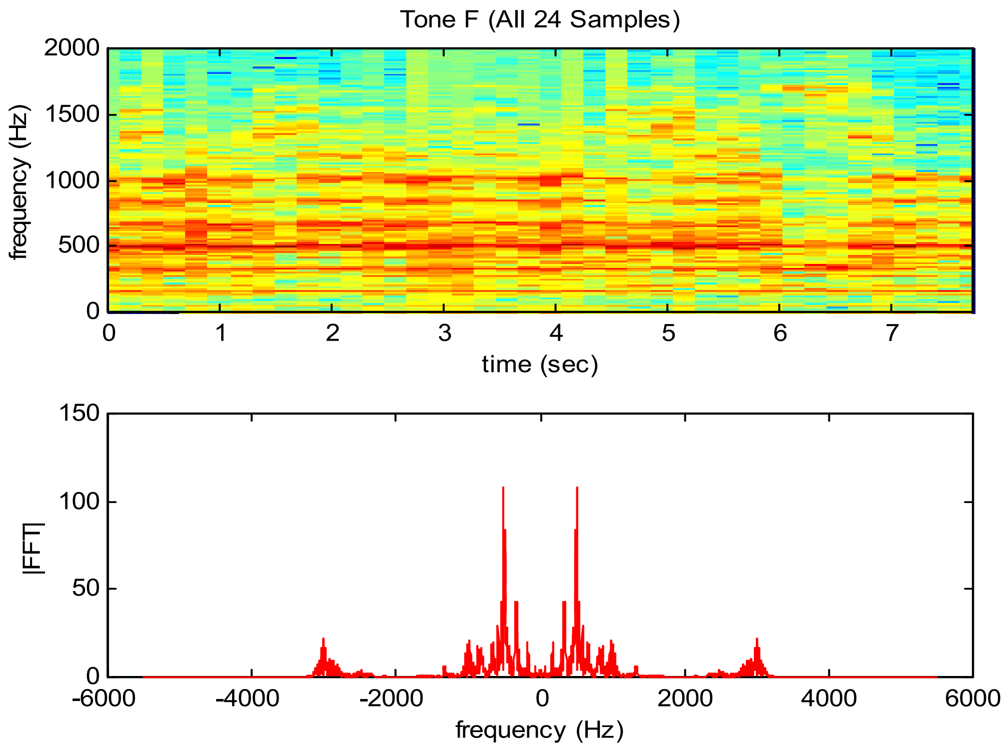

Figure 8-b

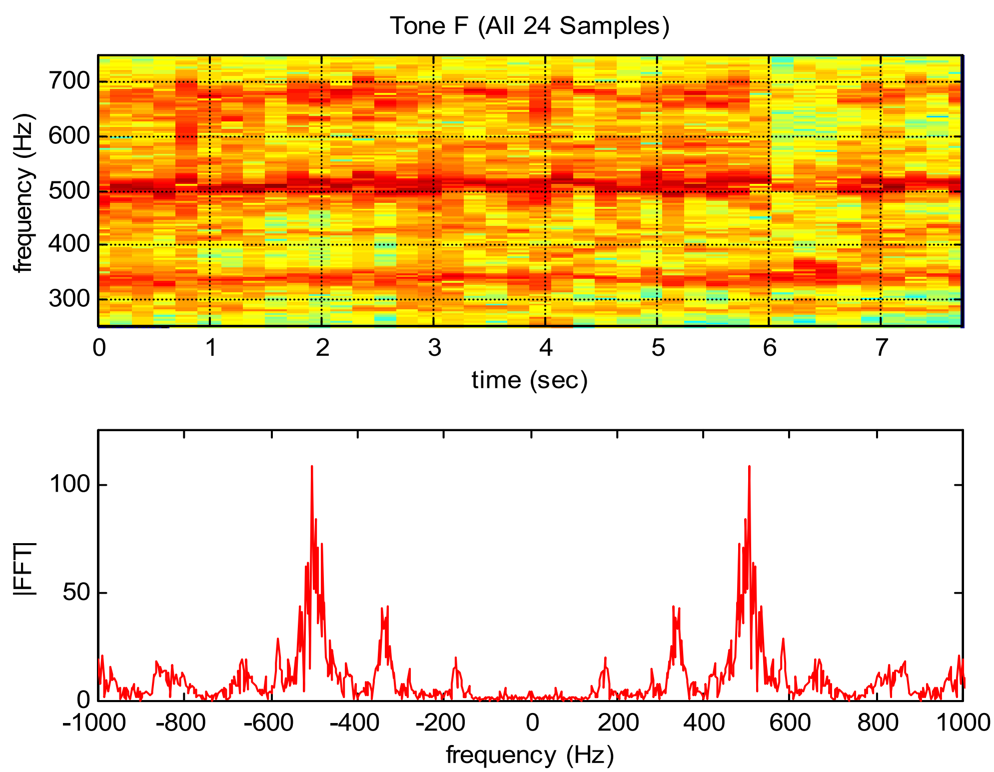

Figure 8-c

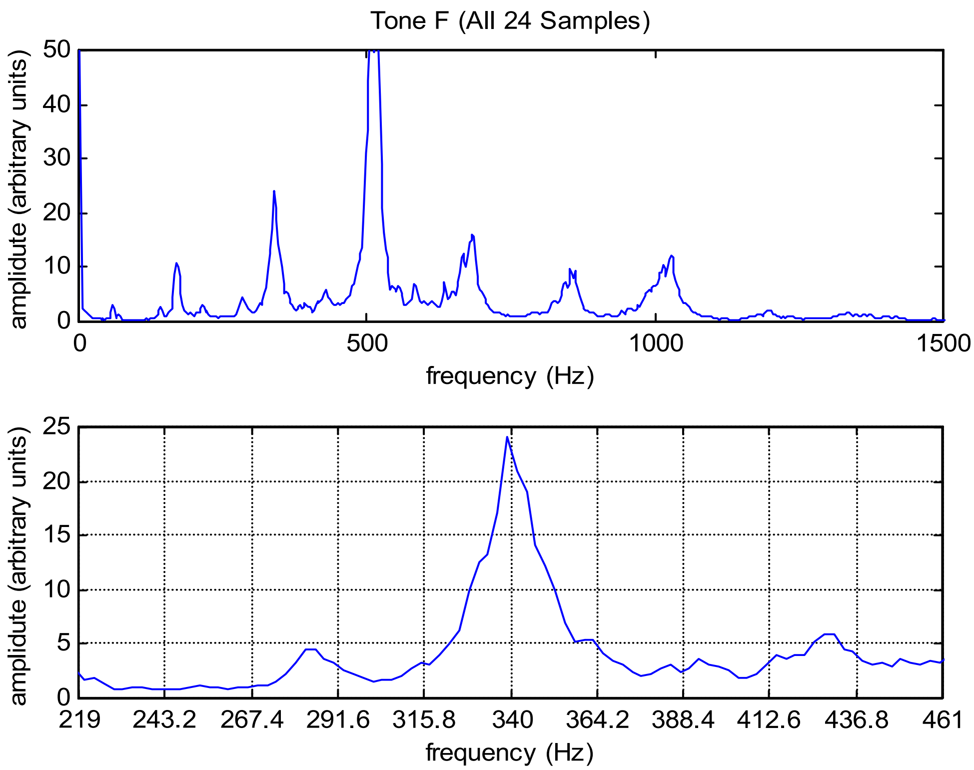

Figure 8-d

The harmonics in figures 8-a and 8-c seem to be stable, compared to figure 6. Since frequency is not fluctuating greatly about the mean, from a first glance it seems that the attraction effect did not take place in tone F case, as expected, because F is a main tone.

Figure 8-d shows the frequency of tone F around 340 Hz. Compared to the accepted value of 349.2 Hz (Jeans, 1968, p. 22) this is not too far off. In order to be certain, however, that we do not have the attraction effect, we need to compare this value to the ones we get when only the possibly pulled tones are treated. Figures 9 and 10 below show these results.

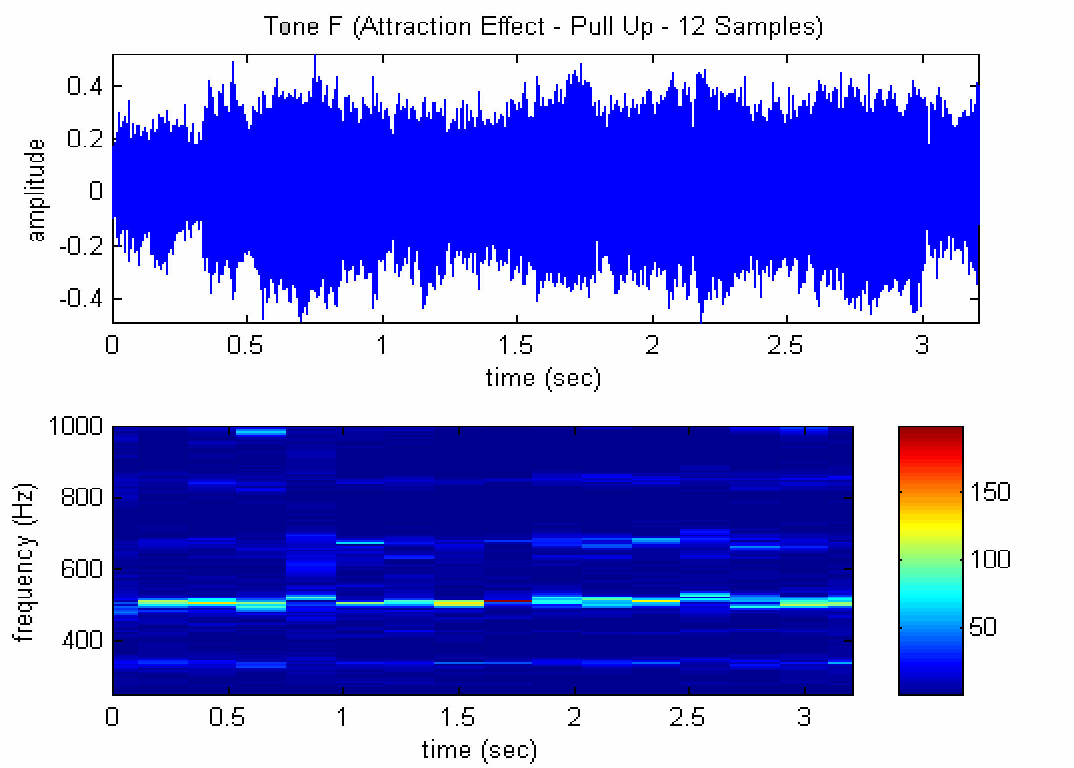

Figure 9-a

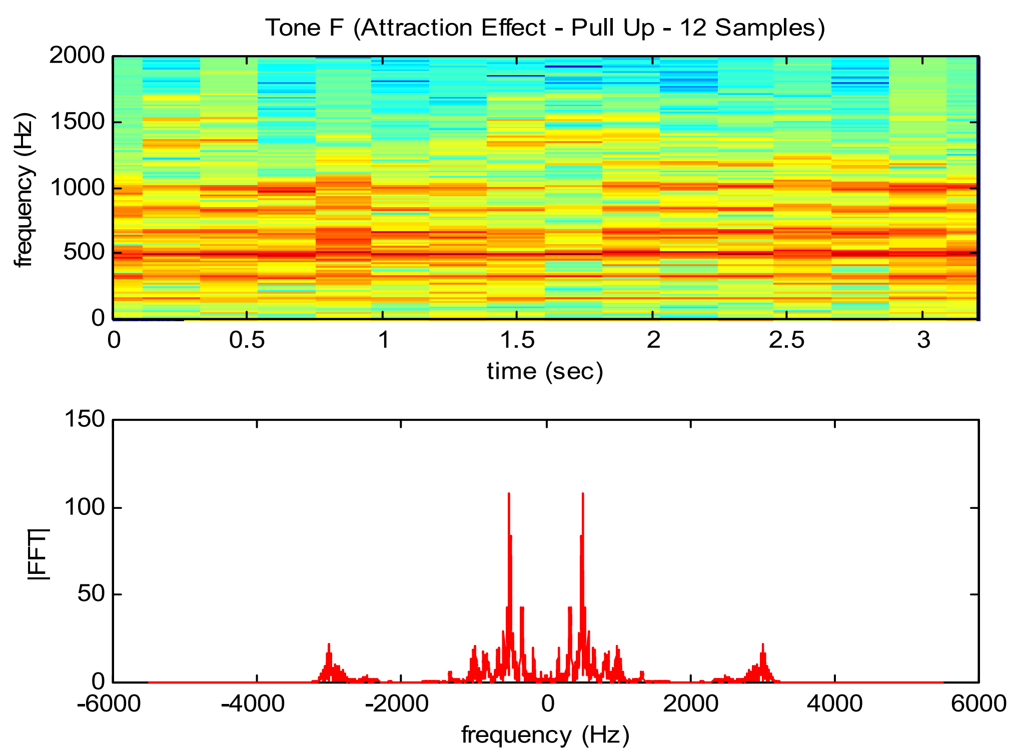

Figure 9-b

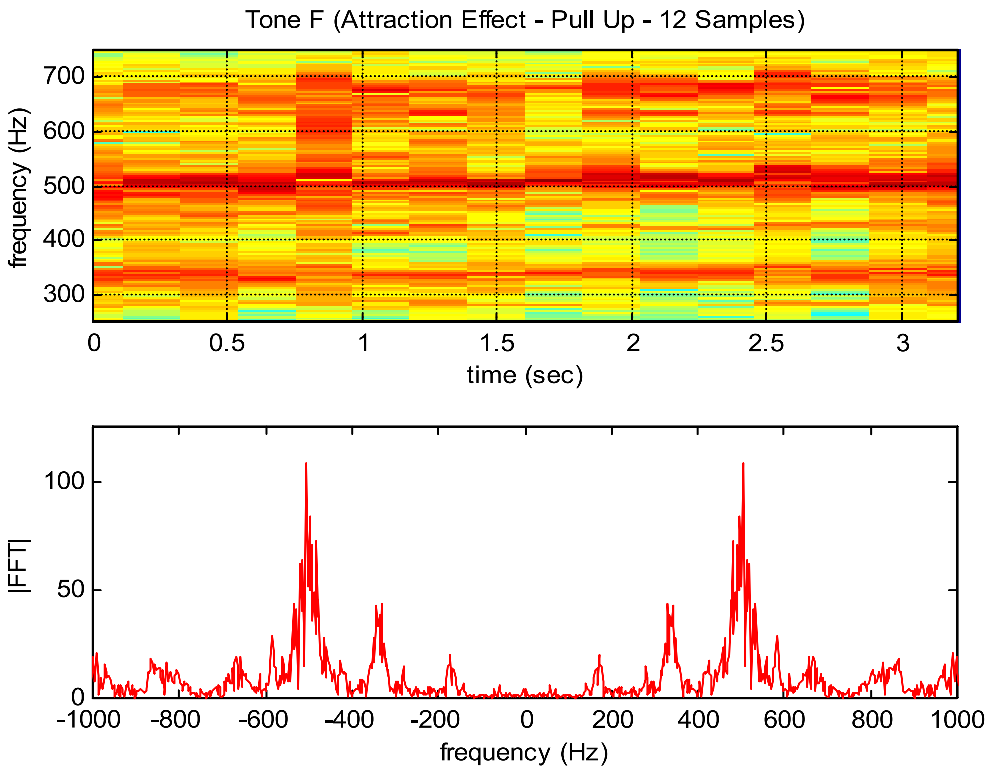

Figure 9-c

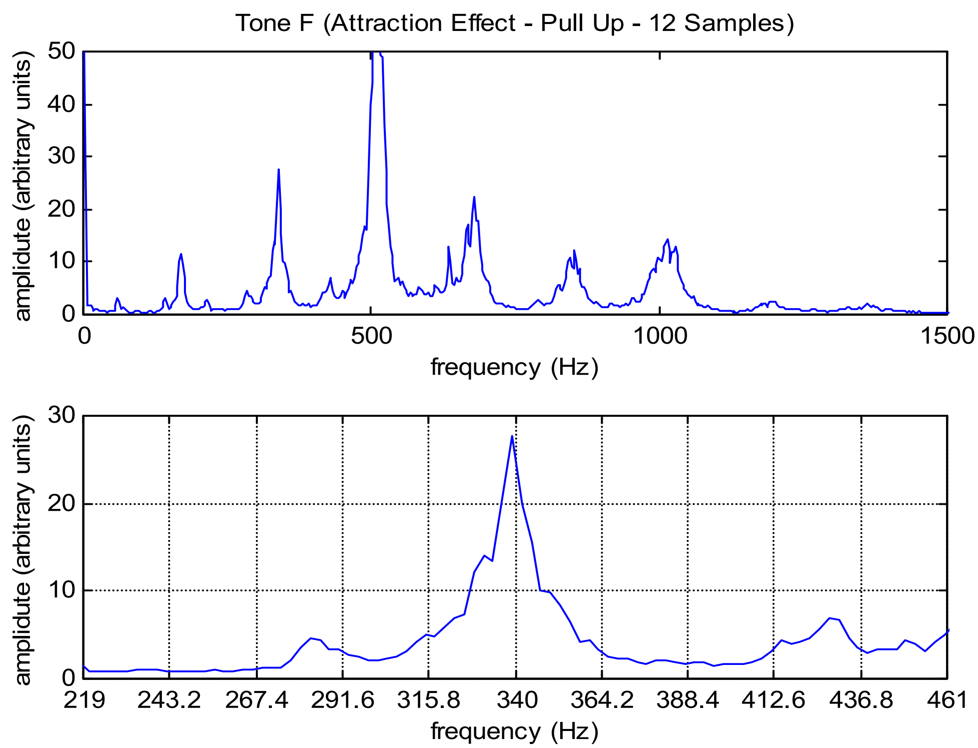

Figure 9-d

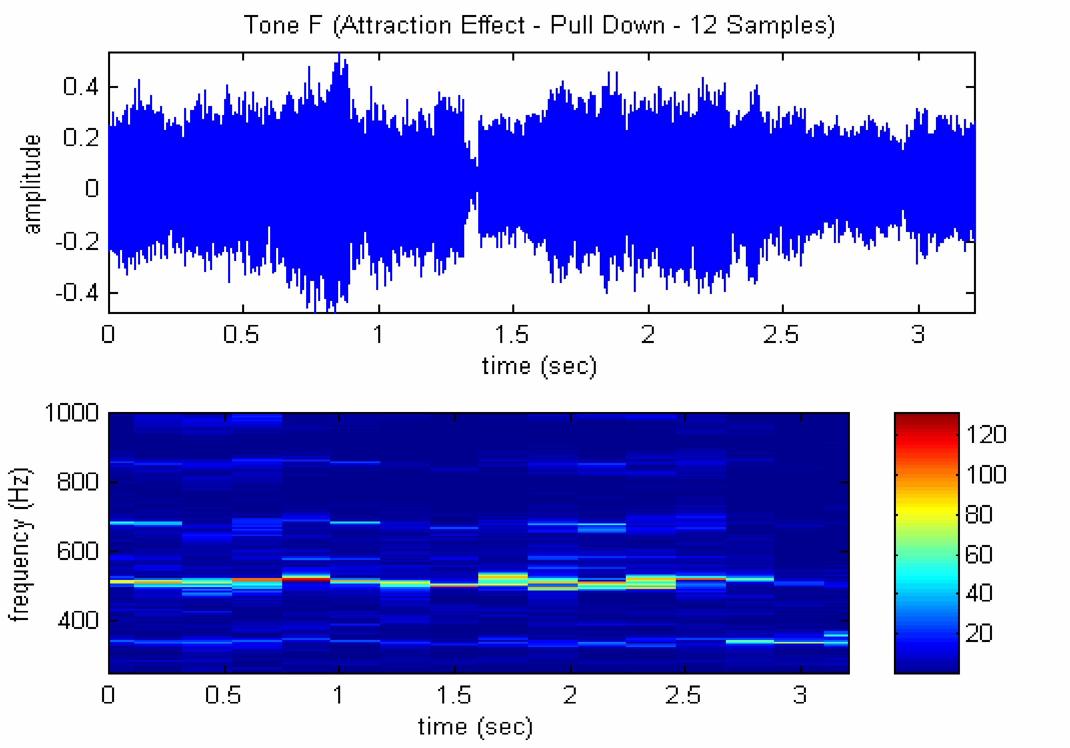

Figure 10-a

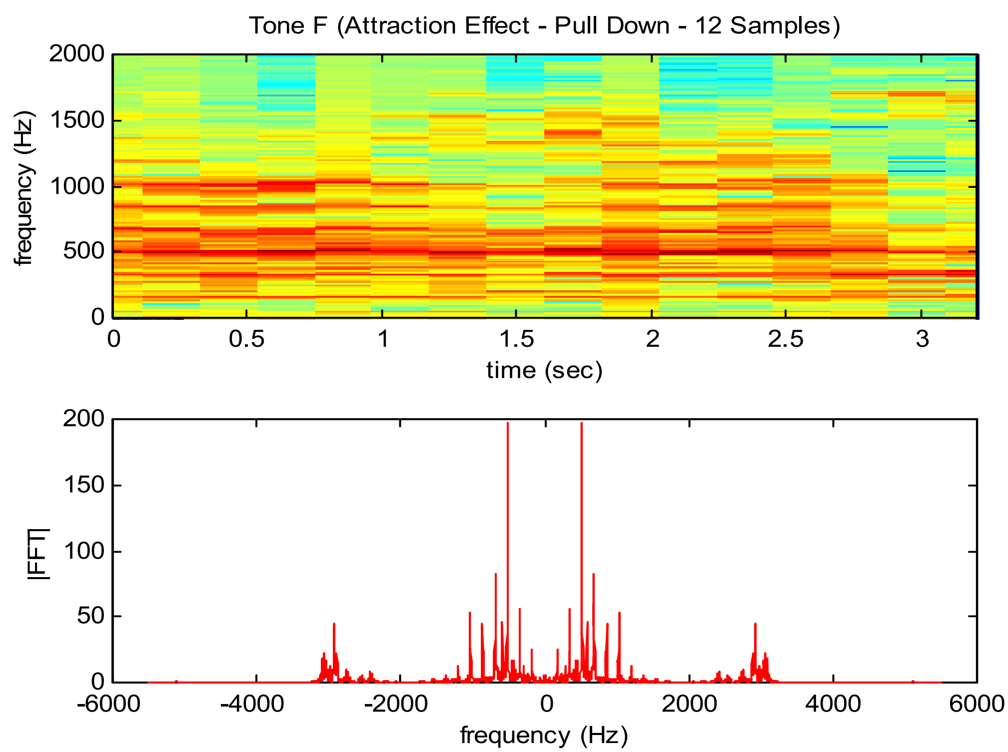

Figure 10-b

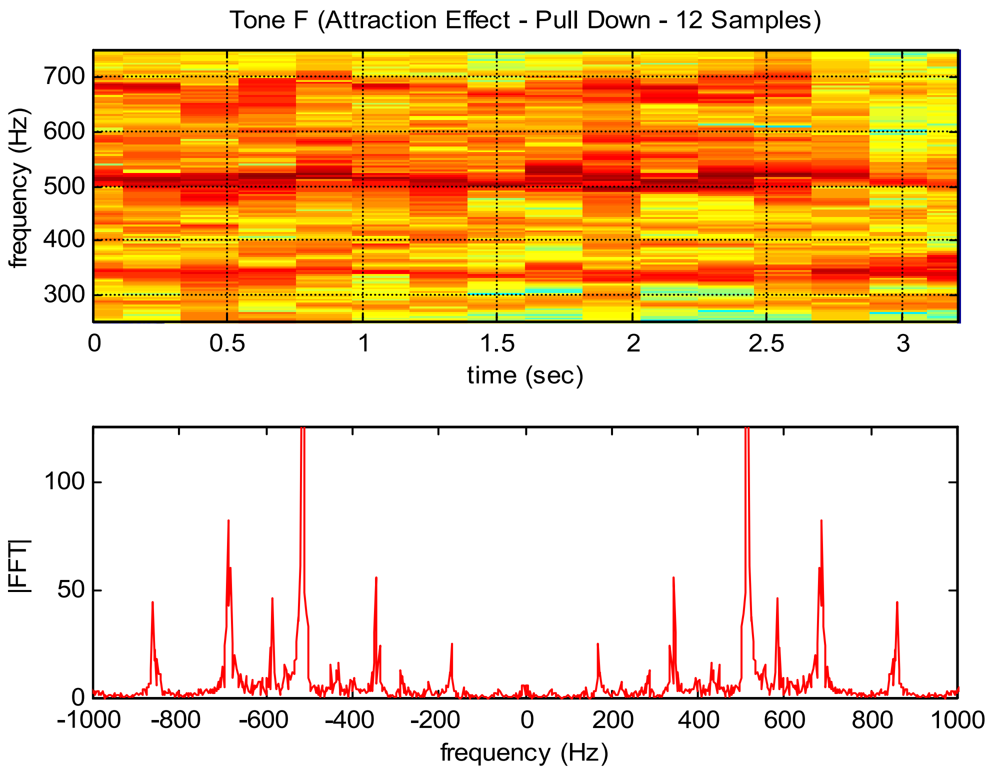

Figure 10-c

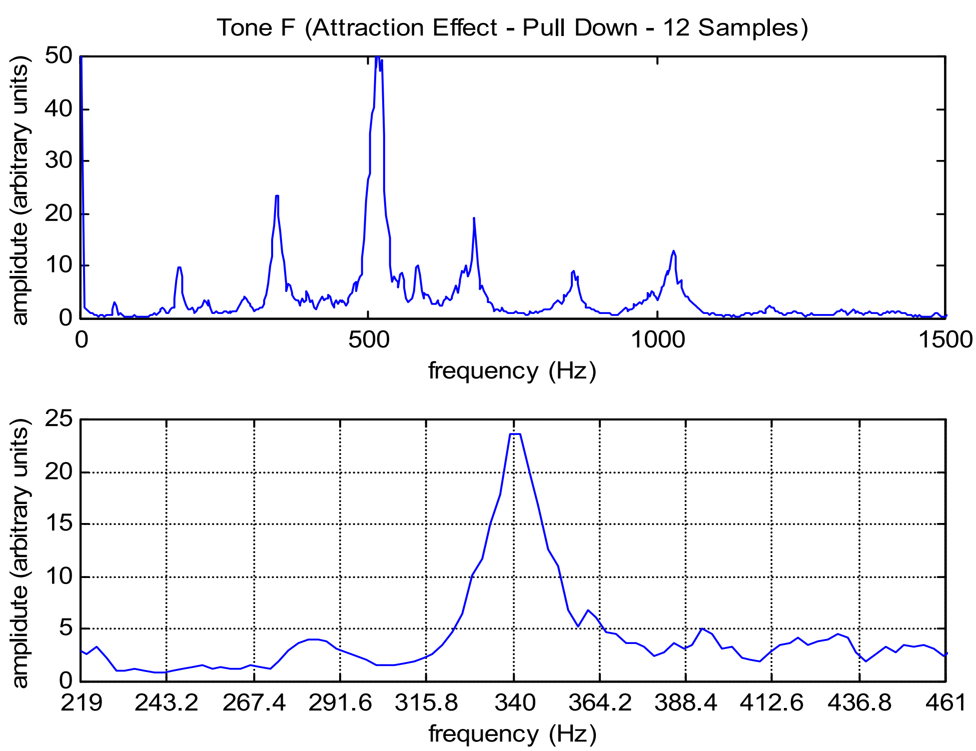

Figure 10-d

Since we see no significant difference in all the three cases shown in this section, we conclude that tone F is not subject to the attraction effect and its frequency is around 340 Hz. We will generalize this result to the rest of the main tones considered in this paper.

SECTION 3-4

TONE G

Tone G is a main tone, therefore the attraction effect is not expected to occur. The accepted value for tone G is 392 Hz (Jeans, 1968, p.22). the results of our analysis are shown in figure 11 below.

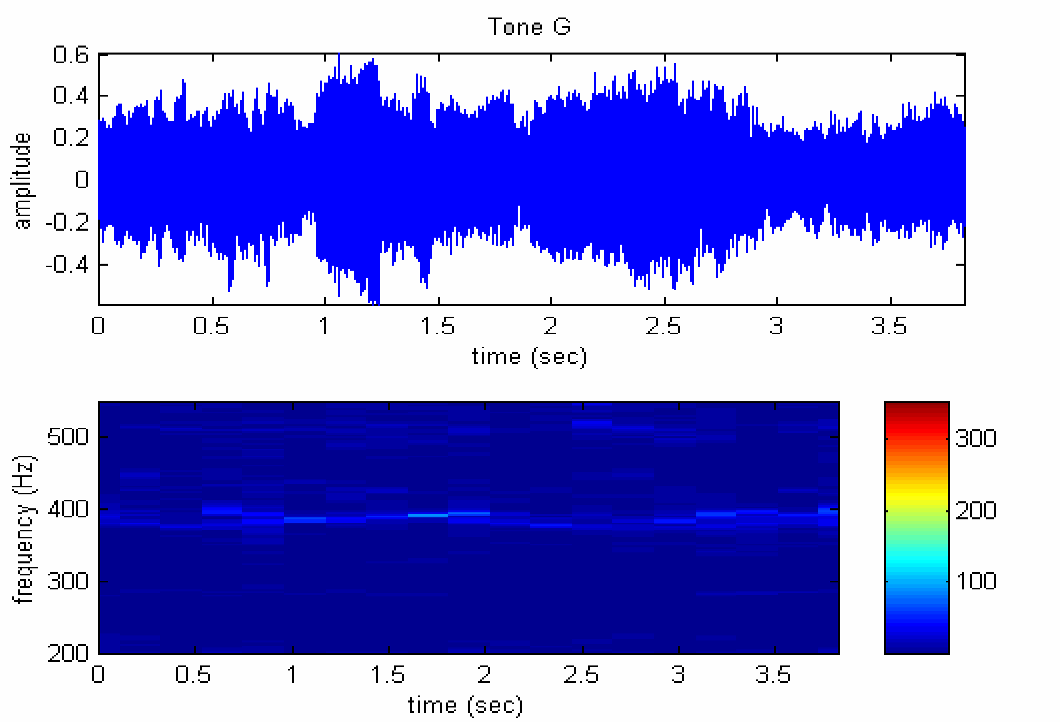

Figure 11-a

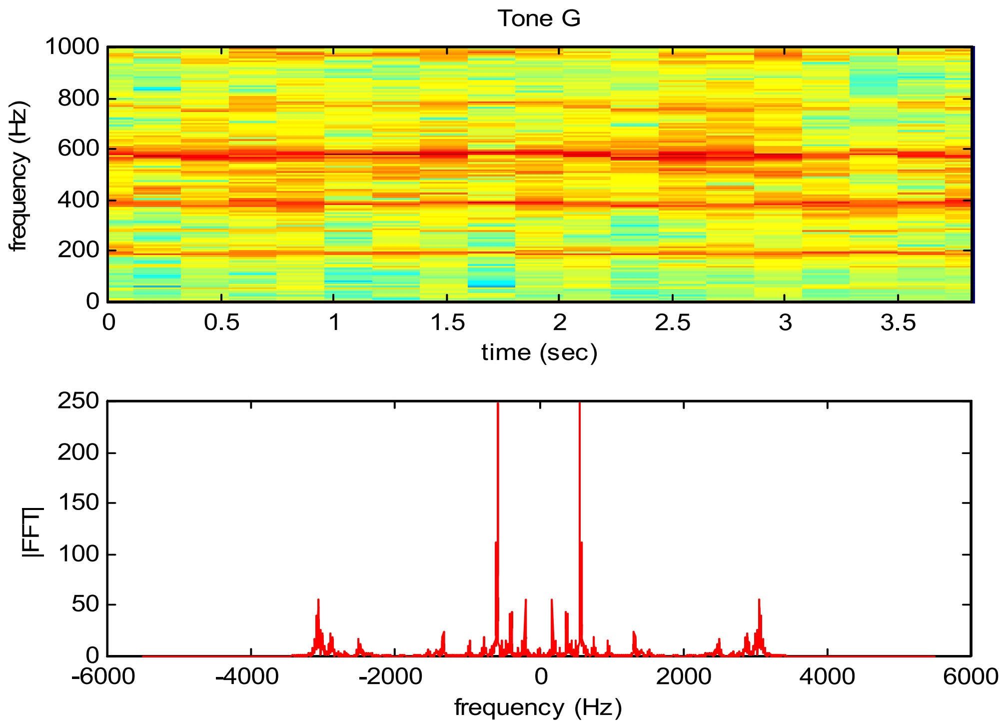

Figure 11-b

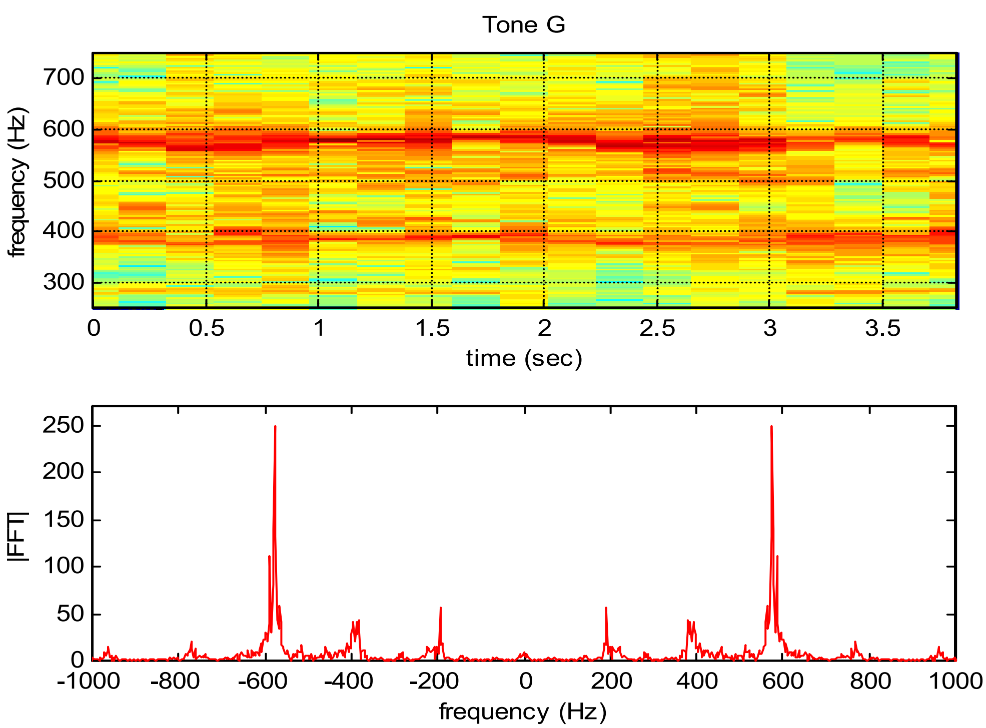

Figure 11-c

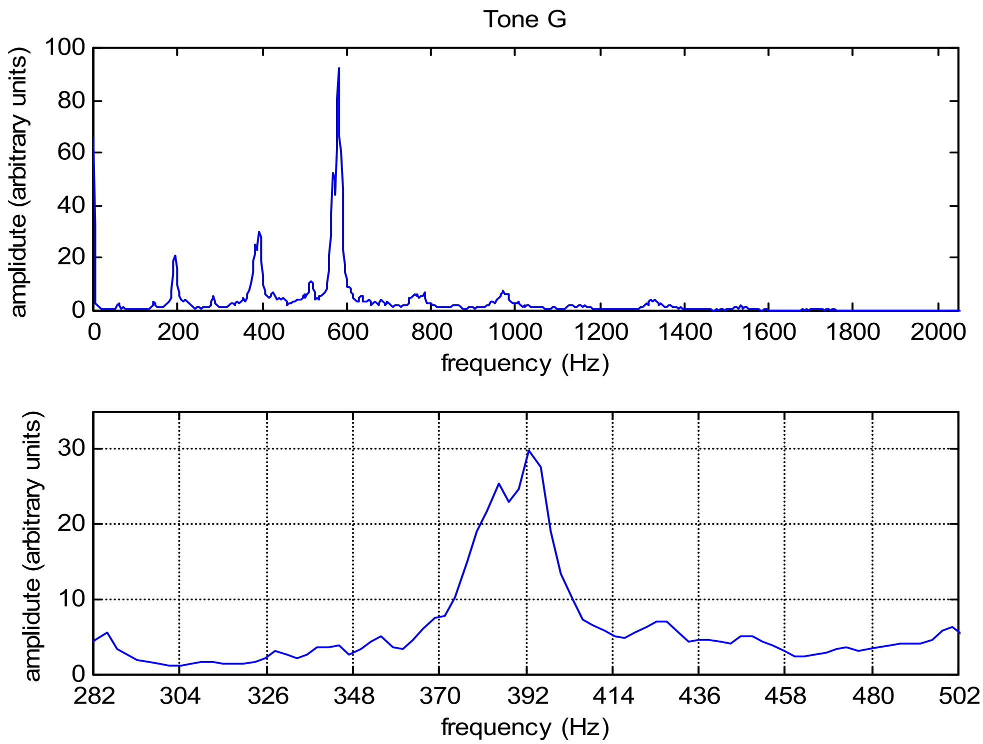

Figure 11-d

The frequencies are stable as seen in figures 11-b and 11-c. Again figure 11-c shows the frequency of tone G. It seems to be at 392 Hz, but because the curve is slightly skewed with a longer tail to the left we will take the value of G to be 390 Hz, which is very close to the European tone G with frequency 392 Hz.

SECTION 3-5

TONE E

Tone E has an accepted value of 440 Hz (Jeans, 1968, p.22) and it was the basis of deriving the frequencies of the rest of the well-tempered scale. It is a main tone and most probably the tuning fork used in this performance was tuned to 440 Hz.

As I said earlier, the music piece selected for this research has many lower tones (first tetrachord), but as one moves up the scale tones become scarcest. Tone A is no exception; for tone A we used 13 samples (snippets). The total time duration of the signal is about 2.6 seconds.

Figure 12 below shows the results. Of particular interest is figure 12-d, which attributes tone A the frequency of approximately 440 Hz. This is pleasantly surprising if we consider that even an analysis on a tuning fork at 440 Hz (figure 2-d) did not yield such clean results. The tone A considered here is similar to figures 1-d and 1-e that show the electronically generated tone analysis.

Consequently, we will take tone A to have a frequency of 440 Hz.

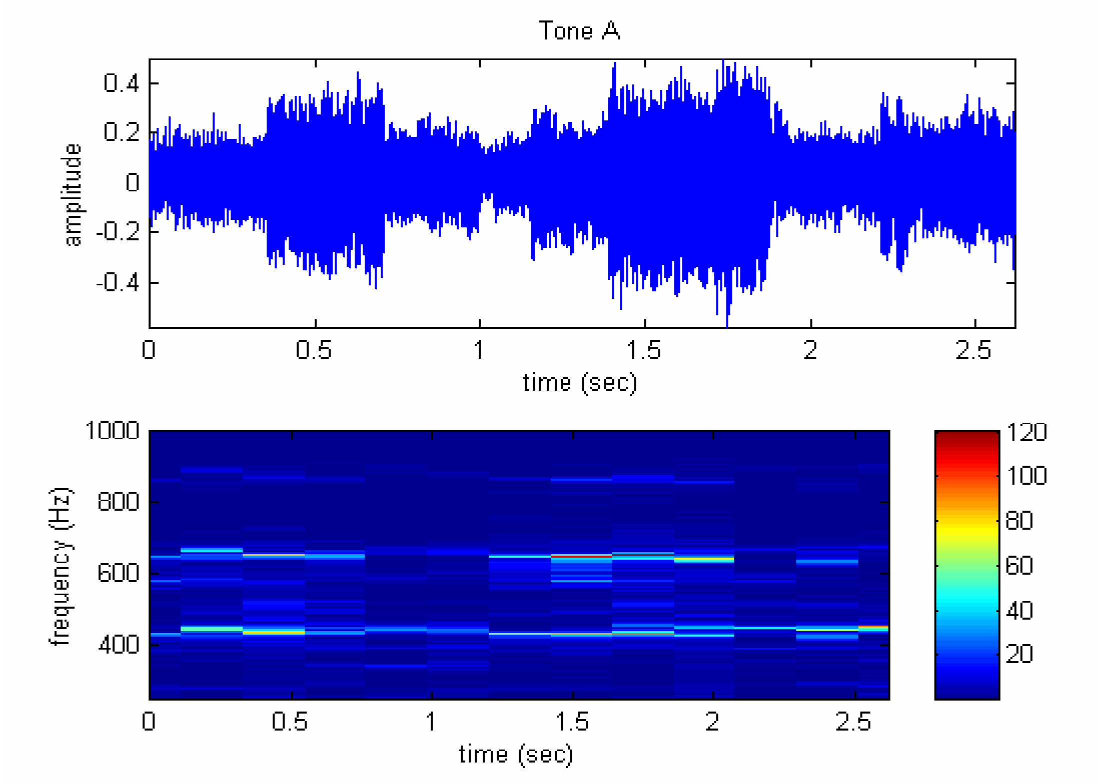

Figure 12-a

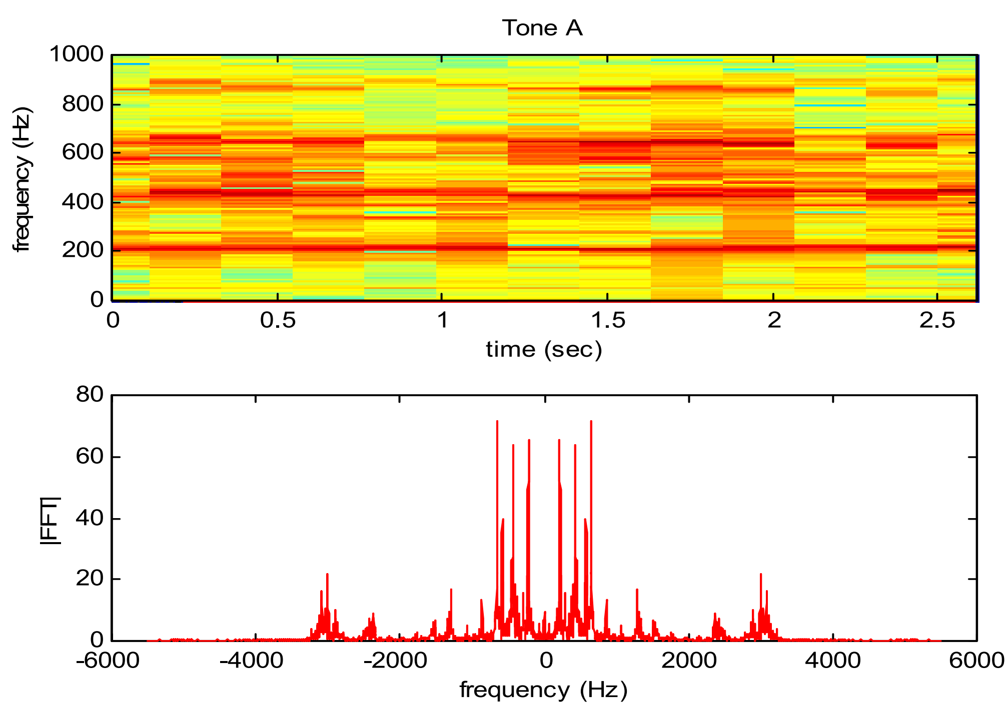

Figure 12-b

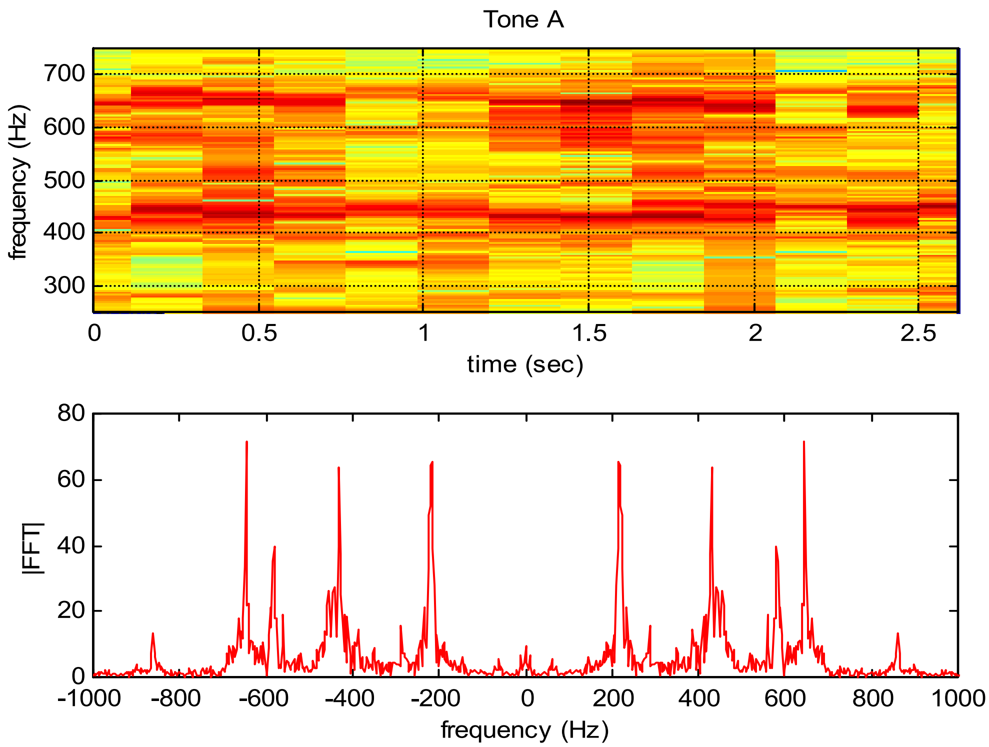

Figure 12-c

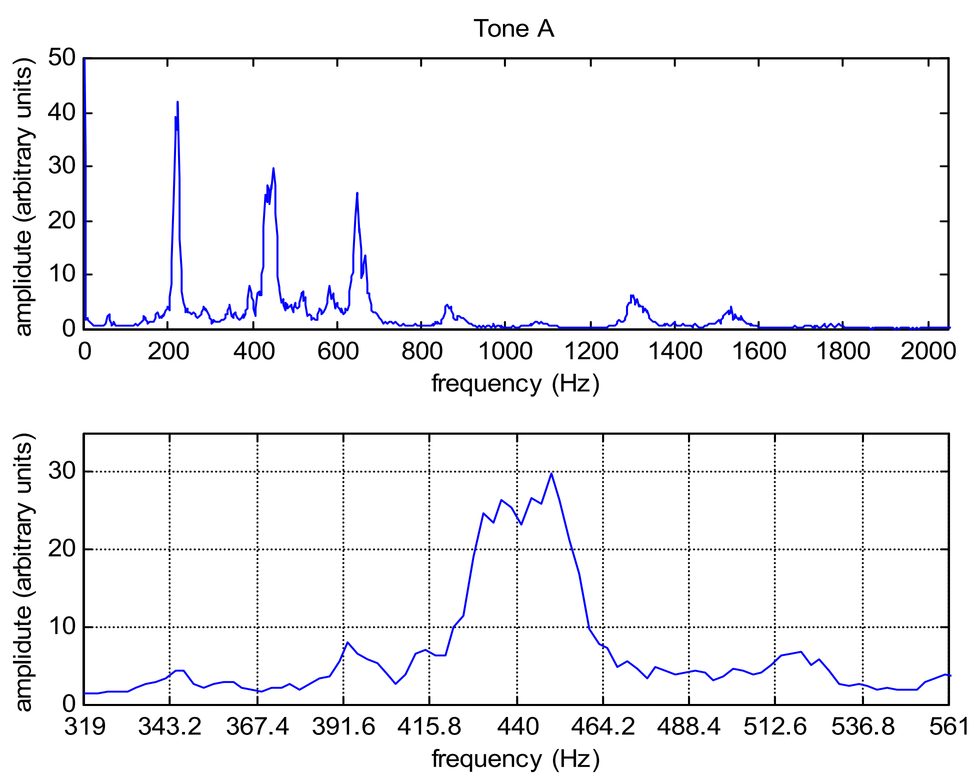

Figure 12-d

CHAPTER 4

DISCUSSION

Frequency Ratios

In this part we deduce the frequency ratios based on our experimental data. Table 3 shows all the results along with the frequency ratios.

TONE	FREQUENCY (Hz)	FREQUENCY RATIO
D	290	1
E_{exp}	313	1.0793
E_{dn}	305	1.0517
E_{up}	320	1.1034
F	340	1.0625 (from E_{up})
G	390	1.1470
A	440	1.1282

Table 3. Frequencies and Frequency Ratios.

The ratios in table 3 are of adjacent tones, i.e., from a variable frequency D one need add 7.93 % to find E experimental. The experimental E tone (E_{exp}) is the one that was derived using both pulled up (E_{up}) and down (E_{dn}) E tones.

When one performs a scale it is customary to sing from the first to the last tone of that scale and then descend from the last to the first. For the diatonic scale, for example, when the chanter sings the ascending part of the scale he performs tones E and B pulled up; accordingly, when he sings the descending part he performs the same tones pulled down. It is peculiar that no known theoretical textbook offers two different diatonic scales, one ascending and another

descending, even though it is well known that the attraction effect is alive and well established in the tradition of the music as well as in its every day use.

Now we attempt to compare the frequency ratios obtained from the scales in figure 1-a and 1-b. To obtain the ratios we make use of the following formula:

$$\begin{aligned} & C \cdot \log_2 \frac{f_f}{f_i} = \text{atoms} \Rightarrow \\ & \Rightarrow \log_2 \frac{f_f}{f_i} = \frac{\text{atoms}}{C} \end{aligned} \tag{14}$$

where f_f is the final frequency and f_i is the initial frequency and the two are adjacent. The base 2 indicates the ratio of the octave (2:1) and C is a constant that denotes 68 atoms (as the total number of atoms in Chrisanth's scale), or 72 atoms in the case of the Patriarchal Committee's scale (figure 1-a). The word "atoms" in equation (14) denotes number of atoms in the boxes of figure 1. Using 68 for figure 1-a and 72 for figure and the appropriate values for Major, Minor and Least tonal intervals we obtain the following frequency ratios for the two scales (Table 4):

	Chrisanth	Patriarchal Committee
Major	1.1301	1.1225
Minor	1.0960	1.1010
Least	1.0740	1.0800

Table 4. Frequency Ratios for the two scales in figure 1.

Using the ratios we can go back and calculate the frequencies of the diatonic scale for the two scales. Then we can compare directly to the frequencies that we found experimentally and see how close theory to practice is. We start from tone A at 440 Hz and we go down to low D and up to high D. Tones B, C and D in table 5 have been constructed using the frequency ratios

of the first tetrachord; these three last tones have not been processed experimentally due to their paucity of occurrence. The results are laid in Table 5. The numbers in italics indicate the frequencies not found experimentally, but calculated from the ratios of the first tetrachord.

TONE	Chrisanth	Patriarchal Committee	Experimental Results
D	292.68	293.67	290
E	320.78	323.33	320 (E_{up})
F	344.52	349.20	340
G	389.35	391.98	390
A	440	440	440
B	482.24	484.44	*485.50*
C	517.92	523.19	*515.83*
D	585.31	587.28	*591.66*

Table 5. Frequencies in Hz for the two scales in figure 1 along with our results. Only tones D to A have been experimentally analyzed.

Notice that both scales use frequency values for E close to E_{up}, which suggests that the inventors of these scales considered only the ascending part of the scale and not the descending. Since it is traditional to give only one scale for both ascending and descending diatonic scales, in this paper we consider only E_{up} as the correct value of tone E. Theoretical textbooks present only one scale, like the one shown in figure 1, as the diatonic scale. I have never come across a theoretical textbook that shows an ascending diatonic scale (with tones E and B pulled up) and a descending diatonic scale next to it (with tones E and B pulled down). It is left upon the guidance of the instructor and the good ear of the student.

The frequencies of the two scales are closely related and are close to the experimental results, except in the cases of tone F. The largest difference in frequency between the two scales is about 9.20 Hz, between the Patriachal Committee's tone F and the experimentally found tone F. Tone F was found substantially lower than both theoretical scales and we reserve this finding

for future research. For the time being, there seems to be no plausible explanation I can give to justify this difference of tone F to both theoretical scales. It could be due to physiological reasons or psychophysical reasons that take place around that specific frequency per se, or it could be a misjudgment of both theoretical scales. We reserve this topic for future research.

Chrisanth's scale seems to be a bit closer to the experimental scale, but again, the frequency difference is so minute it doesn't make much difference to talk about one being closer to the experimental scale than the other.

We can also express the experimental frequencies of Table 5 in terms of atoms for readers that are more comfortable with atoms than frequencies. Since we already have the atoms for the two scales in figure one, we need only find the atoms for the experimentally derived scale. For Chrisanth's scale which has a total number of 68 atoms (figure 1-a) the minor tone has 9.6573 atoms (D – E interval), the least has 5.9475 (E – F interval) and the major has 11.8340 (G – A interval). For the Patriarchal Committee's scale which has a total number of 72 atoms (figure 1-b) the minor tone has 10.2254 atoms (D – E interval), the least has 6.2973 (E – F interval) and the major has 12.5301 (G – A interval). These results compared with the atoms shown in figure 1 also suggest that the differences are minute within the context of pitch discrimination (see p. 79).

Now we turn our attention to another issue: how much a performer is allowed to deviate from these theoretical frequencies. So far we have shown that Mr. Stanitsas is very accurate in performing the theoretically proposed intervals. This argument, of course, is better put in context the other way around. Since chanting relies mostly on tradition and since Mr. Stanitsas is one of the chanters most representative of traditional singing, the theorists did well in assigning the correct number of atoms to each interval. But the theoretical scale doesn't tell us how much you can deviate from a given tone and still be performing the diatonic scale correctly. For this reason

we found the mean, standard deviation and the variance of each tone experimentally using the individual occurrence of each.

Here we make a distinction between what we have been presenting so far (plots) and what we are about to present as the mean of a given tone. So far we have pinpointed the frequency of each tone based on the graph on amplitude vs. frequency of the transform of the concatenated occerrences. Different samples or "snippets" have different time length and therefore different number of points (N). When we did the FFT (and the spectrogram) we used a window on the concatenated data *and* an FFT length of 4096 points. The two lengths were the same and the window that was applied to each snippet didn't center on each snippet perfectly. This could lead to potential leakage, even though it is going to be undersized.

The other aspect of using same window/FFT length is that the mean average frequency is taken over the whole number of snippets. This means that longer snippets are weighted more than the shorter ones. In other words, if we have to average two sound signals, one 1 second long and the other 20 second long, the later will affect our mean more than the one-second-long signal.

We can take each snippet's data matrix and multiply it with a window of the same length as the data matrix, then take the FFT of that product (multiplication in time domain is the same as convolution in the transform domain) and find the peak of that snippet, i.e., its frequency. The longer the FFT length, the more padded zeroes we have and the more accurate is the mean frequency of that snippet. Then we use these frequencies of the, say 20, different snippets of the same tone to find the mean, standard deviation and variance of a given tone. The results are shown in table 6 below.

TONE	Chrisanth	Patriarchal Committee	Experimental Results	Snippet Mean and SD
D	292.68	293.67	290	290.27±3.04
E_{up}	320.78	323.33	320 (E_{up})	320.37±5.57
E_{dn}			305(E_{dn})	309.19±4.61
E_{exp}			313(E_{exp})	314.78±7.59
F	344.52	349.20	340	341.04±4.96
G	389.35	391.98	390	389.20±6.61
A	440	440	440	440.71±7.20

Table 6. Two theoretical values and two experimental values. (SD: Standard Deviation).

The window length was the same as the data (snippet) length and the FFT length has 16384 samples. This gives a $\Delta f \approx 0.6729$ and that is why I keep the numbers in table 6 in two decimal places. Because the snippet length was always significantly smaller than the FFT length, zero padding was applied.

The first comment on table 6 is that the mean frequency obtained from averaging all snippets individually is remarkably close to the value obtained over all realizations. This is an indication of the fact that the concatenated data did not have long snippets far from the mean. Notice, for example, that tone A still possesses its approximately 440 Hz frequency. With the exception of E_{dn}, all other tones are comparatively very close in frequency. How close is close enough we will consider below. First we introduce the necessary basis for our research concept of pitch discrimination and we then proceed to connect it with the standard deviation.

Pitch Discrimination

In the subsection above we found the differences in frequency and we considered them negligible. In chapter 3 we said on various occasions that the frequency difference is not of an order that should alarm us as significant. For example, the pro-echos was lower from the final

termination tone D by 7 Hz, the two scales differ from a fraction of a Hz up to 5 Hz etc. How much is significant or negligible for this research is determined by how much the human ear can resolve. After all, in this paper we are dealing with vocal music, and if a performer cannot differentiate two tones that physically differ in frequency he cannot reproduce this undistinguishable interval. In this section we briefly discuss the psychoacoustic aspect of pitch discrimination.

Pitch is the psychological aspect of sound sensation related to the physical characteristic of a sound, namely its frequency. A sound has a frequency (physical characteristic), but if there is no one there to hear it doesn't posses pitch. Pitch is the physiological aspect of an acoustic wave. Work on pitch discrimination was done extensively in the 1920's and 1930's, even though experiments had been conducted since the 1880's. One of the most sited references is that of Shower and Biddulph (1931), which is described as the most elegant and controlled study on pitch differentiation (Gulick, 1971).

The results show that for frequencies between 125 and 2000 Hz, at a comfortable sensation level of around 40 db the human auditory resolution (Δf) is about 3 Hz (Gulick, 1971, p.126). Some authors refer to this Δf as the *just noticeable difference* or jnd for short. There are other ways one can express this finding ($\Delta f/f$, etc.), but for our purposes the above statement is sufficient.

Throughout this research we came across frequency differences of 5 and 7 Hz. Even though the trained ear can hear a minute difference under ideal conditions, these differences are certainly negligible. Needless to say that any differences less than 3 Hz are nonexistent for our ears.

Within this context of the jnd, a standard deviation equal to jnd, i.e., 3 Hz, is absolutely justifiable; we cannot expect the performer to perform differences that cannot even be heard. Tone D for example, is the one with a standard deviation of about 3 Hz, so the chanter was performing it *every time* very close to the mean value of tone D (~ 290 Hz). This is not surprising since the chanter had the group of accompanying chanters to "remind" him exactly where tone D was (isokratima). By the way, inferences like this one (based on separate snippets) could not be done with the mean frequency obtained from all realizations considered, that is why we averaged the snippets independently.

Standard deviation is a way to define how much a performer can deviate from the indicated theoretical or experimental mean frequency value of a given tone. Since theoretical scales do not provide us with the acceptable frequency deviation for a performance to be accounted as a correct performance interval-wise, we resort to finding this acceptable deviation by experimentation. The more reliable (recognized) the subject (chanter) the more reliable our results would be.

Standard deviations in table 6 vary from about 3 to 7 Hz, therefore we can suggest that a chanter that deviates by about 2 jnd's from the mean frequency of a tone is of the class of Mr. Stanitsas, one of the most recognized chanters.

Is the above acceptable deviation of 2 jnd's the same for main and secondary tones? Notice that standard deviations are not smaller for main tones and therefore a very good chanter should deviate from the mean frequency tone not much more than 2 jnd's.

Is readily seen from table 6 that 2 jnd's is the acceptable deviation for both main and secondary tones. Since we have already established that attraction effect is a real effect naturally occurring in chanting, we conclude that secondary tones subject to the attraction effect are

methodically and systematically reproduced, not by chance, but intentionally. The attraction effect is a voluntary act of performing Byzantine Music scales according to tradition. Since secondary tones usually produce dissonant intervals with respect to the basis of the mode, the chanter must be rather skillful to achieve such an admirable consistency as that of Mr. Stanitsas.

In this last part of the discussion section I will provide evidence that a rigorous mathematical treatment of atoms is not necessary for implementing better interval performance. This will be tied with the fact that Chrisanth may have not treated the subject of atoms extremely scientifically, because he believed that scales are correctly learned given two facts: the teacher knows how to perform the scales correctly and the student has the ability to learn.

Chrisanth stated that the student should learn from a "Hellenic chanter" and not another, because he himself was acquainted with oriental, European, and BM and he knew that other musicians use different intervals. He also stated that "on the schematic representations of the scales [like figure 1], where intervals have numbers of 11 or 13 [atoms], due to the undistinguishable of the unity, they are called major tones [12 atoms] (see introduction p. 18)." In other words he defined 1 atom as his allowed deviation. Let's check this using equation (14). Because we need a frequency ratio that correspond to frequencies that differ by 3 Hz (jnd) we will start from tone D which has 290 Hz and let us see how many atoms is the ratio 293/290. Surprisingly this ratio corresponds to 1.0096 atoms. Chrisanth was correct on defining his jnd intuitively. What about at an octave higher than 290 Hz? The ratio is 583/580, which corresponds to 0.5061 atoms. At the lower limit, let us check around 200 Hz, a sound that very seldom if ever used in chanting. The ratio 203/200 corresponds to 1.5465 atoms.

This last remark will have a special appeal to the reader acquainted with BM. Of course, if we find the atoms according to the Patriarchal Committee's total number of 72 atoms, the

corresponding atoms for the jnd in the various limits of frequency tones are very close to the ones of Chrisanth, as expected. But the Patriarchal Committee never said anything about allowed variations. They devised an instrument to accurately and rigorously determine the intervals, as if they could be performed in such an accurate way.

Chrisanth totally intuitively and long before the jnd was established experimentally gave an allowed deviation from the mean frequency approximately equal to jnd. On a historical note, the first scientific attempt to define jnd some 70 years after Chrisanth's publication, found jnd to be around 0.2 Hz (Luft, 1888), much further from the correctly accepted value than Chrisanth's intuitive guess.

CHAPTER 5

CONCLUSIONS

The result that both scales are approximately equally close in frequency to the experimental results suggests that neither is better in imposing how the intervals of the diatonic scale should be performed. By “equally close” we mean that even though both scales differ from the experimental results, these differences are negligible and occur in both scales. The above argument is better put in realistic context reversely: scales are accurate portrayals of culture.

We experimentally found tone F substantially lower than the tone F proposed by the two theoretical scales. We reserve this topic for further investigation.

How accurate a performance is better realized when observed within the context of two factors: how close are the experimental results to the theoretical scales and how consistent the chanter is within the piece. The experimentally derived scale is close to both theoretical scales considered in this paper, and the chanter is consistent throughout the piece. To examine the first factor we considered an FFT weighted average. For the second factor we used the individual snippets to determine the average and standard deviation.

When standard deviation is tied in context with the notion of jnd’s we define an experimental allowed deviation from the mean, not provided by theory. This result can be used in applications like determining how good is a good performance, according to other performances used as standards.

Representing scales schematically using atoms is no more than a visual aid, since both approximate the experimental results well. A chanter trained by a traditional teacher will perform traditional intervals no matter which scale he is asked to perform. This may not be the case with a singer trained in the well-tempered scale of European music, or a chanter who was trained outside the Byzantine music tradition.

Crisanth's approach to atoms being not as rigorous as that of later theorists is justifiable. Determining intervals in a meticulously mathematical manner does not ensure better performance or a better way of teaching the Byzantine Music scales.

The attraction effect occurs regularly in performing the diatonic scale. Theoretical textbooks choose to represent the diatonic scale using only the ascending portion of it. Pulled tones are taught traditionally. Theorists speak of the attraction effect in a more general form, not indicating the effect on the schematic representations of the scales.

Secondary tones have no more standard deviation than main tones. This suggests that the attraction effect is not only a well established phenomenon in the Byzantine Music tradition, but it is intentional and hard to achieve.

Suggested Future Research

In this paper we considered only one scale (diatonic), two proposed theoretical scales of the same school (traditional) and one performer as sample (Mr. Stanitsas). Therefore our conclusions are limited within the above boundaries.

For future research we will analyze the remaining three scales of Byzantine Music (two chromatic scales, and an enharmonic) and determine the means and standard deviations. We will

do this using a traditional performer and another performer representative of the new movement in Byzantine Music circles. Then we will compare the traditional interval with the modern interval and see how different they are. We will determine if the modern movement intervals are closer to traditional Byzantine Music intervals or closer to European intervals.

We can use different performers representative of the same school (traditional or modern) and see how close they are to each other. We will try and determine if the differences within movements are the same as differences in between movements. If not we establish experimentally that differences exist and we can – to some extent – quantify these differences.

We will see if performers like Mr. Stanitsas are as consistent with other scales as he is with the diatonic. This way we may conclude that some scales are "easier" to perform than others.

Determine if tone F is performed lower than the proposed by the theoretical scale value not only using one music piece performed by one chanter, but using several pieces performed by other representative chanters of both movements (traditional and modern). It is believed that European vocalists prefer the Just scale over the well-tempered scale. A further investigation will show if tone F performed lower is closer to the Just scale, and thus, closer to the preference of European vocalists.

We can conduct experimental work not only with Byzantine Music intervals, but also with the way special characters are performed (qualitative characters). And within this idea we can again check across movements, scales and performers.

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APPENDIX A

Appendix A contains audio files (*.wav) that will help the listener better relate with the research and especially the Methodology section. First we deposit part of the music piece used to extract the snippets. Then we provide the concatenated snippets as one can hear them when running the appropriate Matlab® *.m file. The numbers (names of wave files) correspond to the description given below.

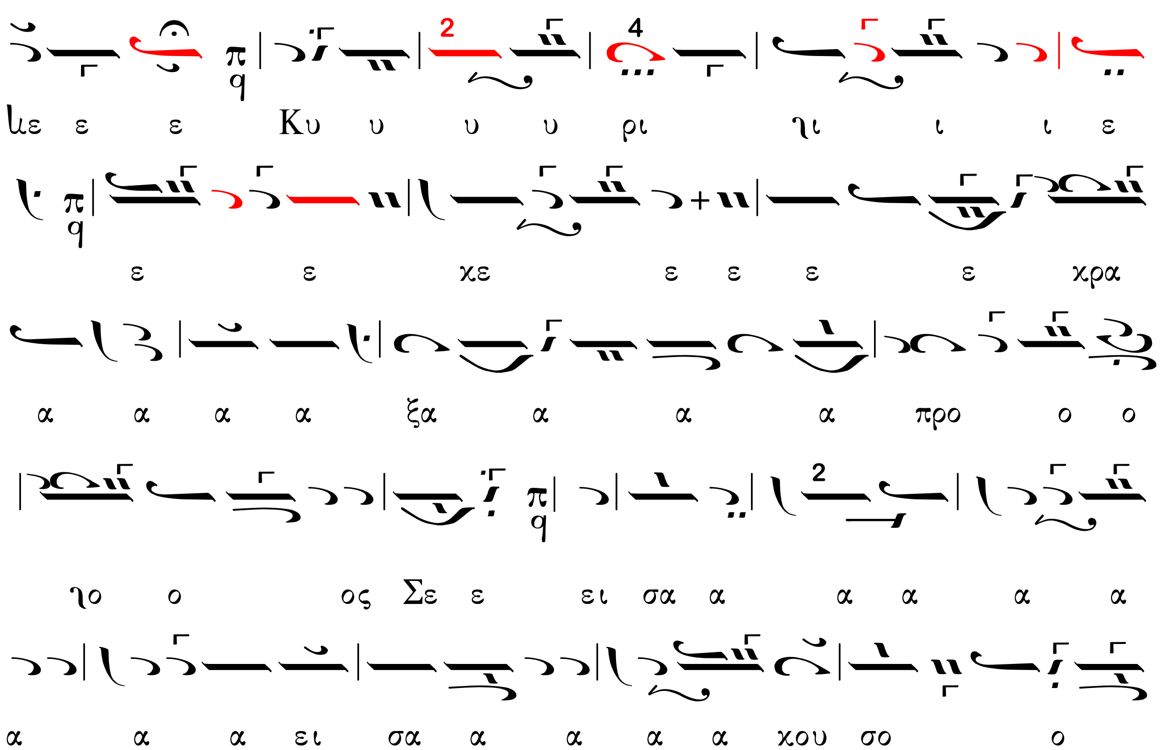
1. Part of Music piece by Mr. Stanitsas.
2. Concatenated D tone.
3. Concatenated E tone with all 40 samples.
4. Concatenated F tone with all 24 samples.
5. Concatenated G tone with 20 samples.
6. Concatenated A tone with 20 samples.
7. Pro-echos.

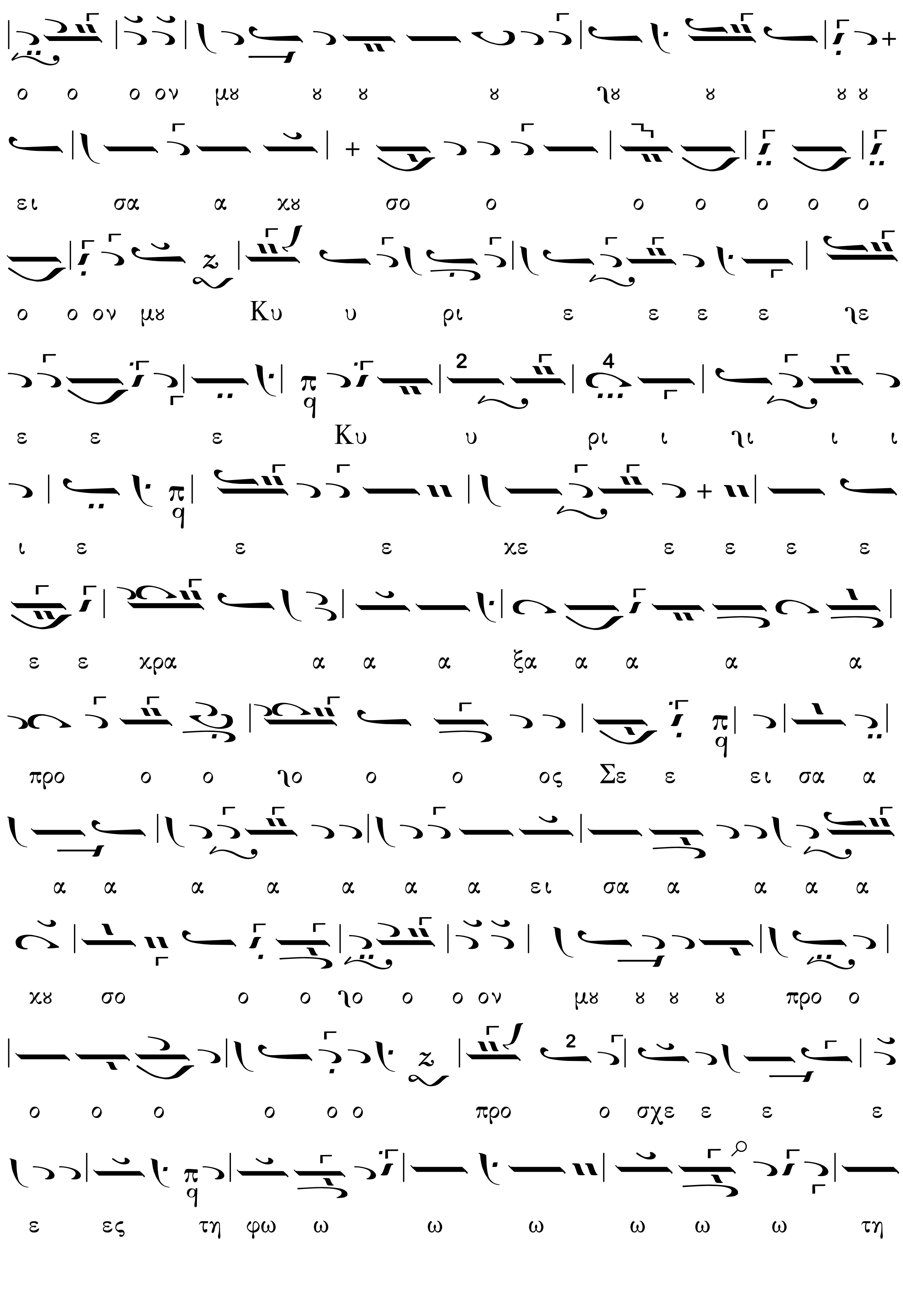
APPENDIX B

Appendix B shows the manuscript of the music piece performed by Mr. Stanitsas. Red characters indicate tone D, but do not necessarily correspond to the correct order of snippets of tone D. Only the first few are shown here in red. This piece was composed by Jacob the arch-chanter and it was kindly provided by Dr. Nick Giannoukakis as a handwritten manuscript. It was typed by the author.

Κεχραγάριον Ἀργὸν
Ἰακώβȣ Πρωτοψάλτȣ (†1800)
Διασκευασθὲν παρὰ Γ. Πρωγάκη

Ἦχος Α΄. Πα

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VITA

Kyriakos Tsiappoutas was born in Cyprus in 1977. He finished the Lanition A Lyceum in his hometown Limassol, in 1995. After completing his military service as a sub lieutenant in the Greek and Cypriot National Army (1997) he enrolled at the University of New Orleans in 1998. In 2002 he completed his Bachelor of Science in Psychology with a minor in Physics and in 2004 he completed his Master of Science in Applied Physics. Both degrees were earned at the University of New Orleans.